Normalisation in Coulomb excitation experiments

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4TH GOSIA WORKSHOP

Outline:

1. Normalisation constants in GOSIA:
   - independent normalisation,
   - user-given normalisation constants.

2. Possible techniques
   (elastic scattering, known lifetimes, target excitation …),

3. Selected applications.

Intro

- Low-energy Coulomb excitation with heavy ions (or high-Z targets) → sensitive tool to probe collective nuclear structure.

- Used in conjuction with complementary spectroscopic data → give us a wide range of information on the electromagnetic properties of nuclei, leading to the knowledge of the **nuclear shape** (nuclear charge distribution).

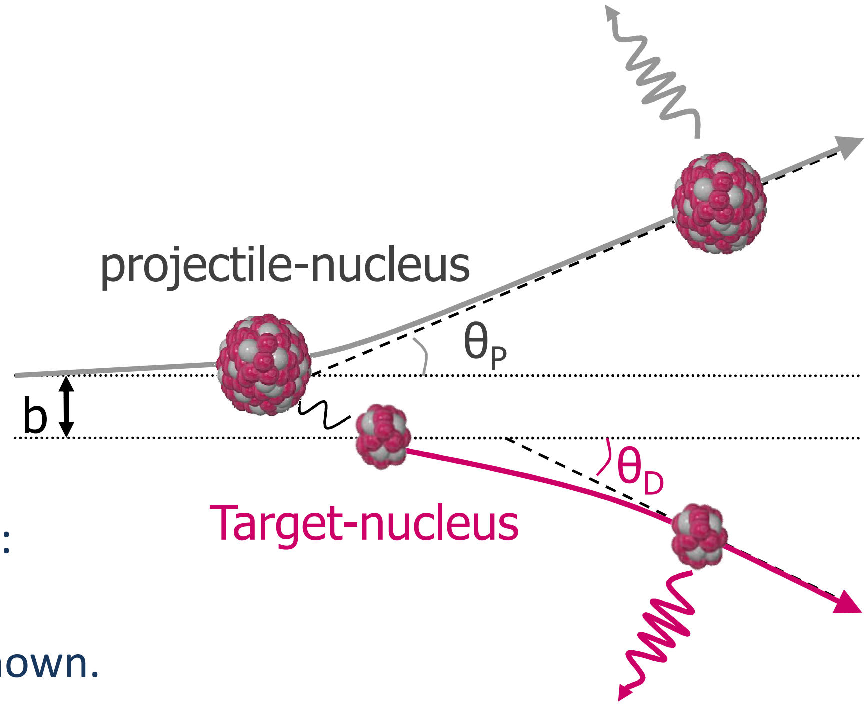
- New challaneges emerge when studying exotic nuclei with Coulomb excitation → low statistics, often lack of complementary data, i.e. precise information on the lifetimes of excited states (especially for short-lived, neutron-rich nuclei).

- New solutions for the normalisation for the measured Coulomb-excitation cross-sections needs to be applied.

Normalisation of measured Coulomb excitation cross sections

Both **excitation** and the consequent **γ-ray de-excitation,** governed by the **very same set of matrix elements,** are calculated within the GOSIA code, allowing for a direct comparison to experimental data.

Normalisation factors - why needed ?



- To convert **measured** γ-ray intensities

  to **absolute** excitation cross-sections

  of the populated states.

- Possible complications arise from the fact that:
  deadtime, beam intensity, efficiency of
  the particle detection set-up, etc… not well known.

- To deal with this GOSIA uses normalisation constants to relate experimental and calculated intensities.

- It is **not possible to impose an absolute normalisation** in the standard GOSIA.

Normalisation constants used in GOSIA

- The normalisation constant *C* fitted to all measured γ-ray intensities I^e

$$\sum_i (C I_i^c - I_i^e)^2 / \sigma_i^2$$

normalisation constant for a given experiment

calculated γ-ray intensity

experimental γ-ray intensity for the *i*-th measured transition

experimental uncertainty

the product of the:

→ Rutherford cross section,

→ absolute efficiency of particle detection set-up,

→ solid angle covered by the particle detection setup

Normalisation constants used in GOSIA

- relative normalisation constants C_m to link *m* experimental data sets (→ different scattering angle, target, etc…)

$$\sum_m \sum_i (C_{\mathrm{global}} C_m I_i^c - I_i^e)^2/\sigma_i^2$$

relative normalisation constant for each *m* data sets

calculated γ-ray intensity

experimental γ-ray intensity for the *i*-th measured transition

experimental uncertainty

- C_m can be **specified by user** or **fitted** by GOSIA.
- C_{global} extracted in the minimisation proccess.

χ^2 in GOSIA

$$\chi^2 = \sum_{i=1}^{N\,\mathrm{exp}} \sum_{j=1}^{N\,\mathrm{det}} w_{ij} \sum_{k(ij)}^{N\,\gamma\mathrm{exp}} \frac{1}{\sigma_k^2} \left(C_{ij} I_k^c - I_k^e\right)^2$$

$$+ \sum_{d} w_d \sum_{n_d} \frac{1}{\sigma_{n_d}^2} \left(D_{n_d}^c - D_{n_d}^e\right)^2$$

$$+ \sum_{m\,(i\,j)}^{N\,\gamma\mathrm{calc}} \left(\frac{I_j^c(i,j)}{I_n^c(i,j)} - u(i,j) \right)^2 \cdot \frac{1}{u^2(i,j)}$$

weights ascribed to the various subsets of data

normalisation constants (equivalent to C_m from previous slide)

spectroscopic data points (lifetimes, BR, mixing coefficients …)

"observation limit" of γ-ray intensities (fraction of normalising transition)

Possible techniques of normalization (1/2)

Normalisation constants can be either **specified by user** or **fitted independently.**

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Several techniques possible, the choice depends on the specific of the experiment

→ normalisation constants defined by the user based on:

elastic scattering, target excitation

(*will be discussed later …*)

Possible techniques of normalization (1/2)

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Several techniques possible, the choice depends on the specific of the experiment
→ normalisation constants defined by the user based on:
elastic scattering, target excitation
(*will be discussed later …*)

GOSIA calculates the best normalization factors (i.e. the ones providing the minimum value of $\chi 2$ for a given set of matrix elements) for each single γ detector independently.

Option **INR** (**I**ndependent **N**o**R**malization) in the GOSIA input file

(more details → see page 122 in the GOSIA manual:

http://www.pas.rochester.edu/~cline/Gosia/index.html)

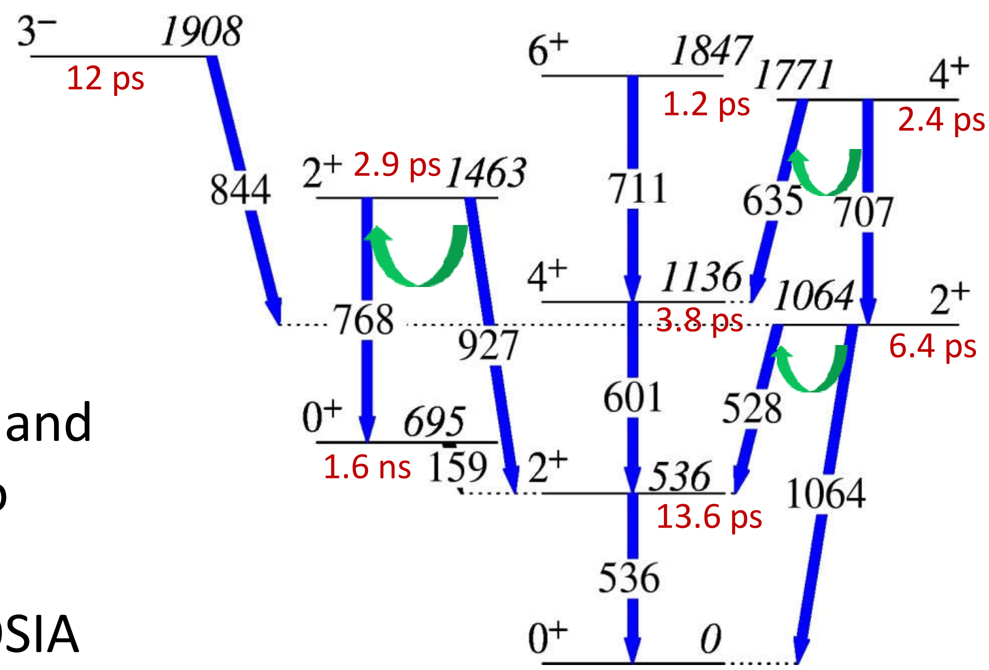
Independent NoRmalization (INR)

- When should be used ?
- Always if possible !

⇩

Number of fitted matrix elements and experimental data large enough to neglect the impact of introducing few more parameters fitted by GOSIA

Multistep Coulomb excitation
(with stable beams, intensities ~ 10^9pps)



^{100}Mo

Coulomb excitation, HIL, Warsaw, 2007
K. Wrzosek-Lipska et al., PRC 86, 064305 (2012)

INR and knowledge of spectroscopic data

- Multistep excitation – one or more B($E2$; $I_i \rightarrow I_f$) values can be used to fit the C_m.
- Observation of the corresponding population of the I_i state is required, i.e., the relevant γ-ray intensity and efficiency, along with the branching ratio, need to be known to good precision.
- This is simplest and preferred method ! → everything is fitted by the code.

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➢ In **even-even nuclei** normalisation is usually fulfilled by an **independent lifetime measurements** → examples for exotic nuclei are the cases of:

^{74,76}Kr E. Clement *et al.,* PRC **75**, 054313 (2007),

$^{182\text{-}188}$Hg N. Bree *et al.,* PRL **112**, 162701 (2014), K. Wrzosek-Lipska *et al.,* EPJA (2019) 55:130

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➢ For **odd-mass** or **odd-odd nuclei multipole mixing ratios** of the γ-ray transitions become important since the strongest-observed γ ray is often a mixed E2/M1 transition .

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➢ Low-energy transitions in **heavy nuclei** can also be **strongly converted**

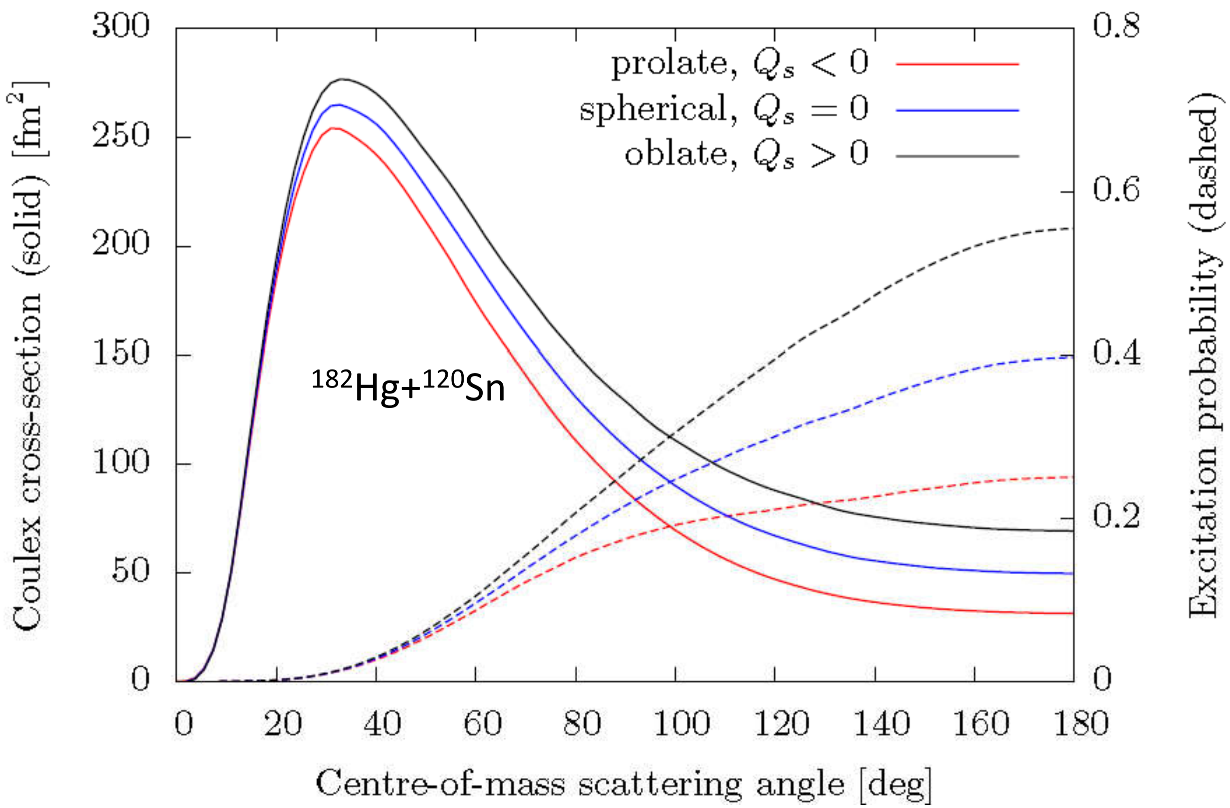
→ strongest excitation path may not necessary result an intense γ-ray decay.

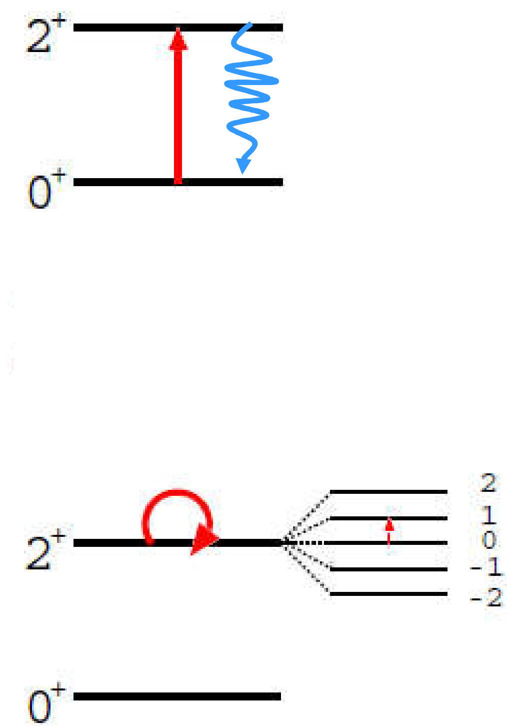
→ normalisation to the next higher-lying transition usually possible:

^{224}Ra L. P. Gaffney *at el.,* Nature **497**, 199 (2013).

Coulomb excitation of exotic nuclei

- lack of complementary experimental data: τ, BR, δ(E2/M1)
- beam intensities rather low: particle detectors at forward angles to maximise the statistics
- low statistics, usually one- step or two-step excitation and only one gamma line observed





$\langle 2^+ || E2 || 2^+ \rangle \sim Q_s$

$$\frac{d\sigma_{clx}}{d\Omega} = \frac{d\sigma_{Ruth}}{d\Omega} \cdot P(i \to f) = \left(\frac{a}{2}\right)^2 \frac{1}{\sin^4(\frac{\vartheta_p}{2})} \cdot P(i \to f)$$

$$P_{2_1^+} \propto |\langle 2_1^+ || \mathrm{E2} || 0_1^+ \rangle|^2 \cdot \left(1 + \langle 2_1^+ || E2 || 2_1^+ \rangle \cdot K(\theta, \mathrm{E}_p)\right)$$

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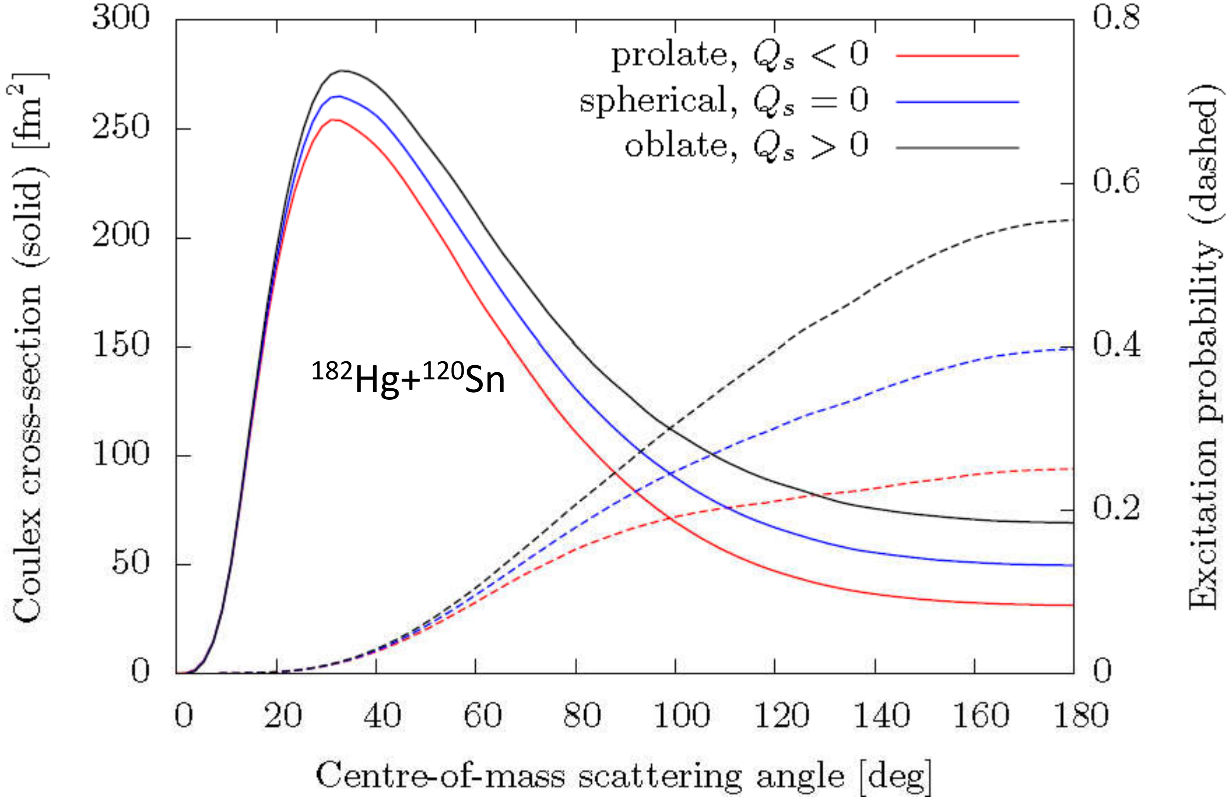

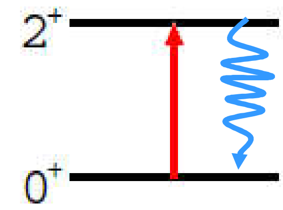

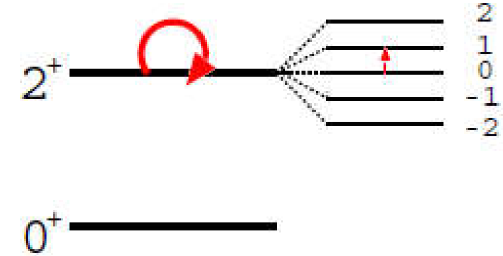

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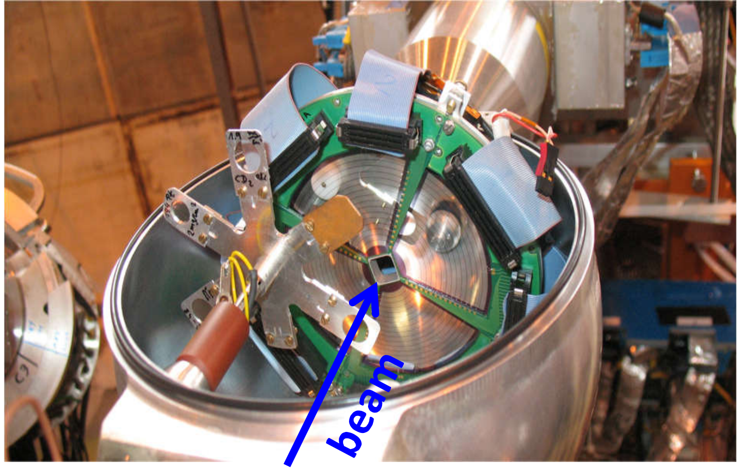
Relative normalization (1/2)



Coulomb excitation with RIBs:

- very limited number of experimental data
- better to avoid introducing additional free parameters (normalization constants)

Relative normalisation of a number of data sets corresponding to different angular ranges based on:

1. **Elastic scattering**
2. **Normalisation to the target excitation**

These normalisation constants (C_m) are specified by user.

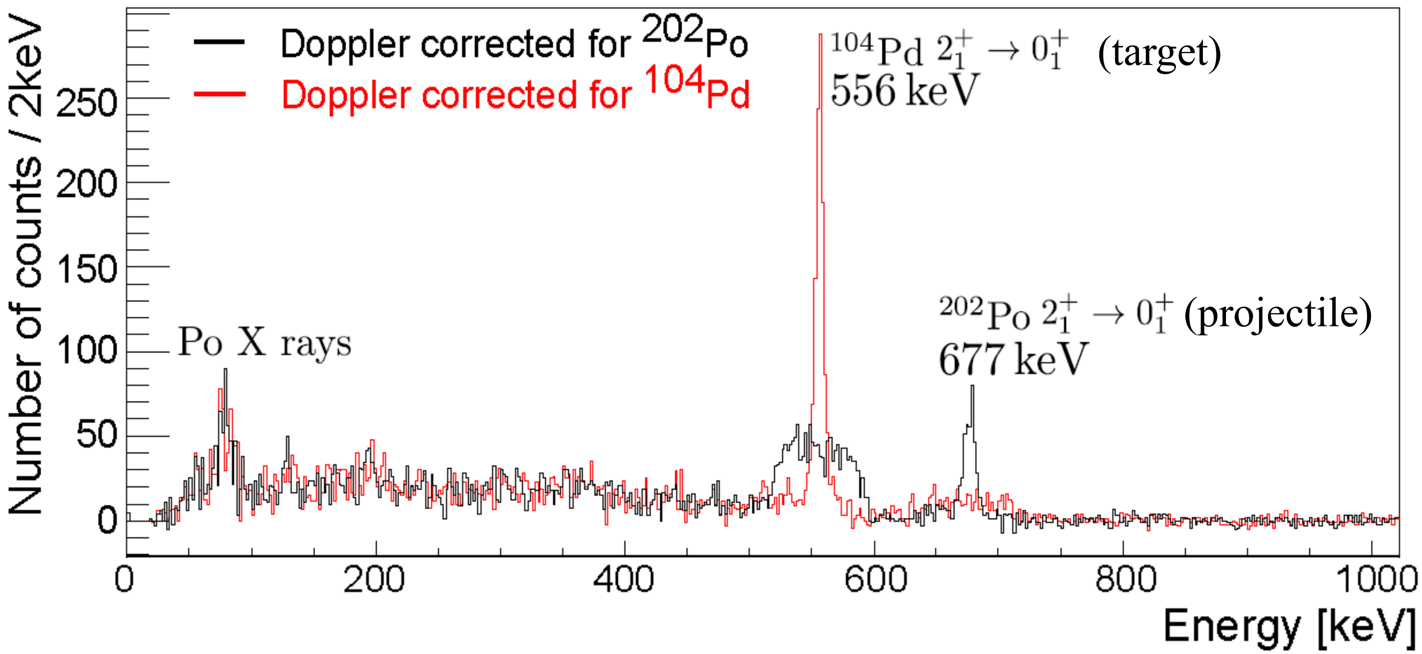
Elastic-scattering (Rutherford) cross-section – historically the simplest and most direct method.

however,…
precise knowledge of the scattering angular range, well understood dead time, beam current is required…

moreover,…

normalisation to elastic-scattering requires other than particle-γ trigger → dowscaled particles need to be measured as well.

Relative normalization (2/2)



Nele Kesteloot,
PhD thesis, KU Leuven,
PRC 92, 054301 (2015)

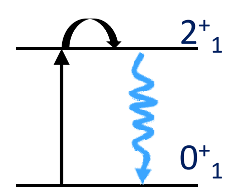
- one-step excitation of ^{202}Po

spectroscopic data not known without other kind of normalization

– impossible to obtain solution

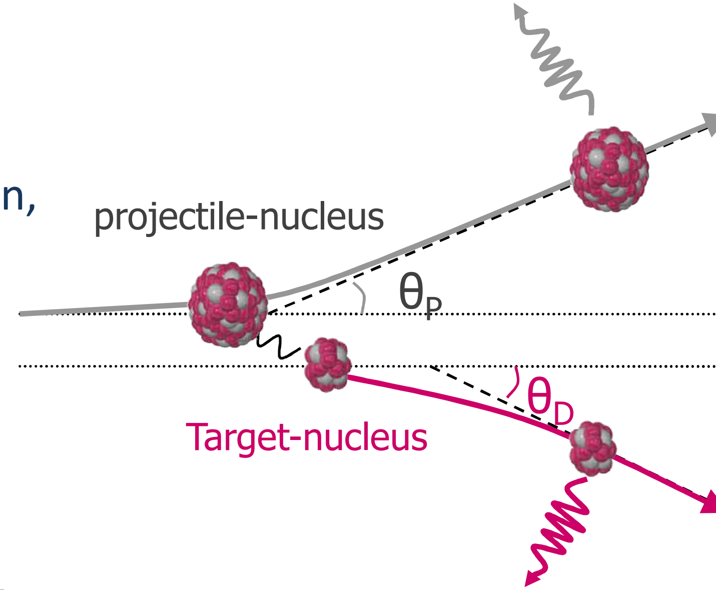
(a modification of the relevant matrix elements can be easily compensated by adjusting the normalisation constant)

any combination of BE2 ↔ Q (2^+_1) will reproduce the γ yield



Normalization to the target excitation

Target excitation



1. Several conditions when choosing a target for the RIBs Coulomb excitation (e.g. kinematic separation, gamma rays overlaping).

2. One of them → electromagnetic structure (*B(E2)*s, *Qs*) of the target nucleus is known.

3. The observed **excitation of the target** can be described with the literature values of *ME*s and used to **normalise** the **excitation cross sections** for the **projectile**

The observed number of γ rays in the transition de-exciting an excited state in the **target nucleus** →

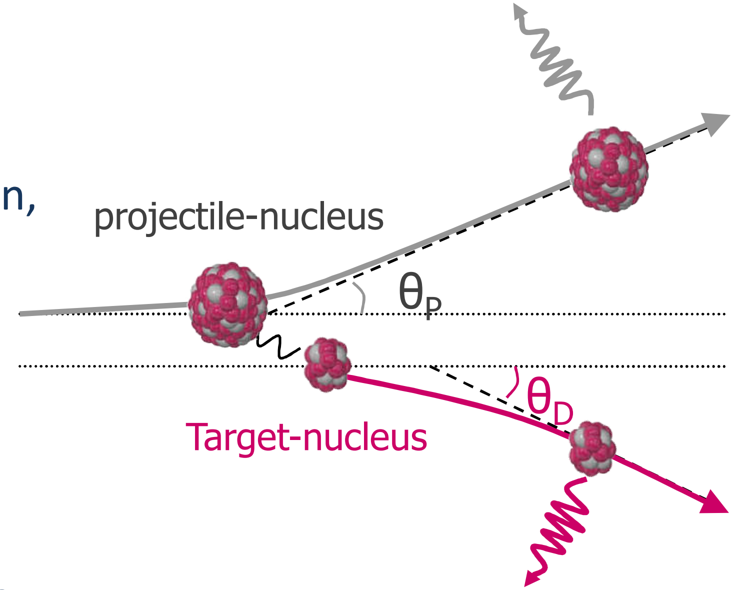
$$N_t = L \cdot \frac{\rho d N_A}{A_t} \cdot b_t \epsilon_\gamma(E_t) \epsilon_{\text{part}} \sigma_t$$

time-integrated luminosity of the beam

total g-ray branching ratio for the transition

integrated cross-section of exciting given state in the target

Target excitation



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The observed number of γ rays in the transition de-exciting an excited state in the **projectile nucleus** →

$$N_p = L \cdot \frac{\rho d N_A}{A_t} \cdot b_p \epsilon_\gamma(E_p)\epsilon_{\text{part}}\sigma_p$$

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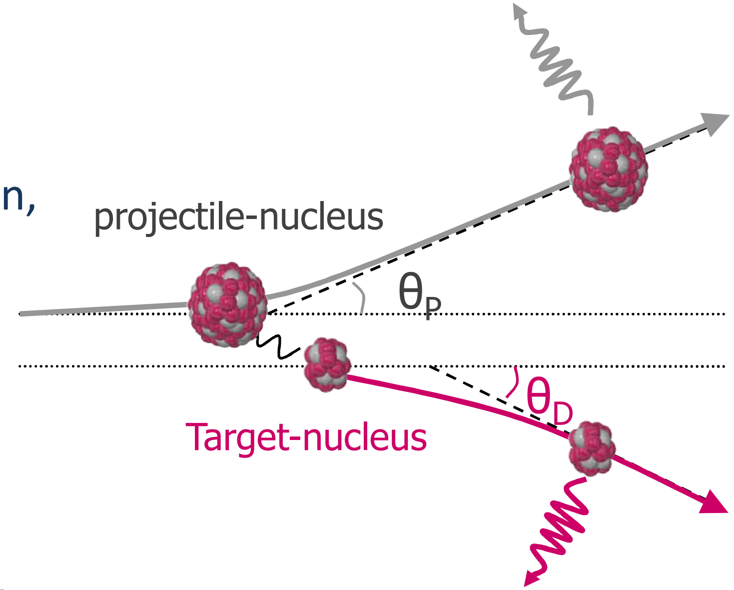


2. One of them → electromagnetic structure (*B(E2)*s, *Q*s) of the target nucleus is known.

3. The observed **excitation of the target** can be described with the literature values of *ME*s and used to **normalise** the **excitation cross sections** for the **projectile**

The observed number of γ rays in the transition de-exciting an excited state in the **target nucleus** →

$$N_t = L \cdot \frac{\rho d N_A}{A_t} \cdot b_t \epsilon_\gamma(E_t) \epsilon_{\mathrm{part}} \sigma_t$$

- L: time-integrated luminosity of the beam
- b_t: total g-ray branching ratio for the transition
- σ_t: integrated cross-section of exciting given state in the target

$$\frac{N_p}{N_t} = \frac{b_p \epsilon_\gamma(E_p) \sigma_p}{b_t \epsilon_\gamma(E_t) \sigma_t}$$

The observed number of γ rays in the transition de-exciting an excited state in the **projectile nucleus** →

$$N_p = L \cdot \frac{\rho d N_A}{A_t} \cdot b_p \epsilon_\gamma(E_p) \epsilon_{\mathrm{part}} \sigma_p$$

Normalisation to the target excitation – GOSIA2

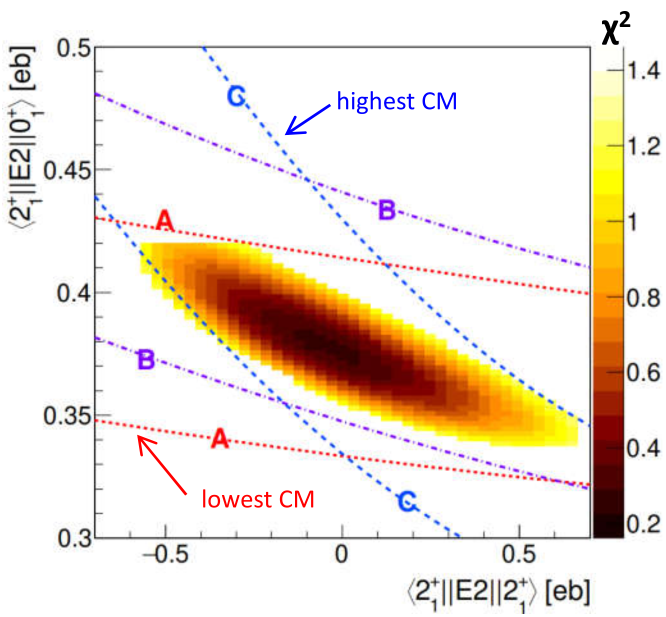
- developed to handle the simultaneous analysis of both target and projectile excitation;
- limited to one combination of beam and target (available at www.slcj.uw.edu.pl/gosia);
- two input files have to be prepared: one for target, one for beam;
- GOSIA2 minimises χ^2 function for the target (this includes calculation of normalisation factors) and then uses the same normalisation factors as a starting point when it starts minimising χ^2 for the beam;
- normalisation factors are shared as parameters across both χ^2 functions and after several iterations best set of normalisation factors found;
- for high CM angles - diagonal matrix element for the target important

$$eQ_{sp} = \sqrt{\frac{16\pi}{5}} \frac{1}{\sqrt{2I+1}} (I, I, 2, 0 | I, I) \langle I \| \hat{M}(E2) \| I \rangle$$

Limitation of GOSIA2

- data collected on more than one target
- error calculation not incorporated – „by hand"
- if one-step excitation for both target and projectile, one can use standard error progression (contributions from:
    - uncertainty of target γ-ray yield
    - uncertainty of projectile γ-ray yield
    - uncertainty of the B(E2) of the target)
- if several angular ranges and quadrupole moment important – χ^2 surface

M. Zielinska The Euroschool on Exotic Beams, Vol. VI, Lecture Notes in Physics 1005, Springer Nature Switzerland AG 2022

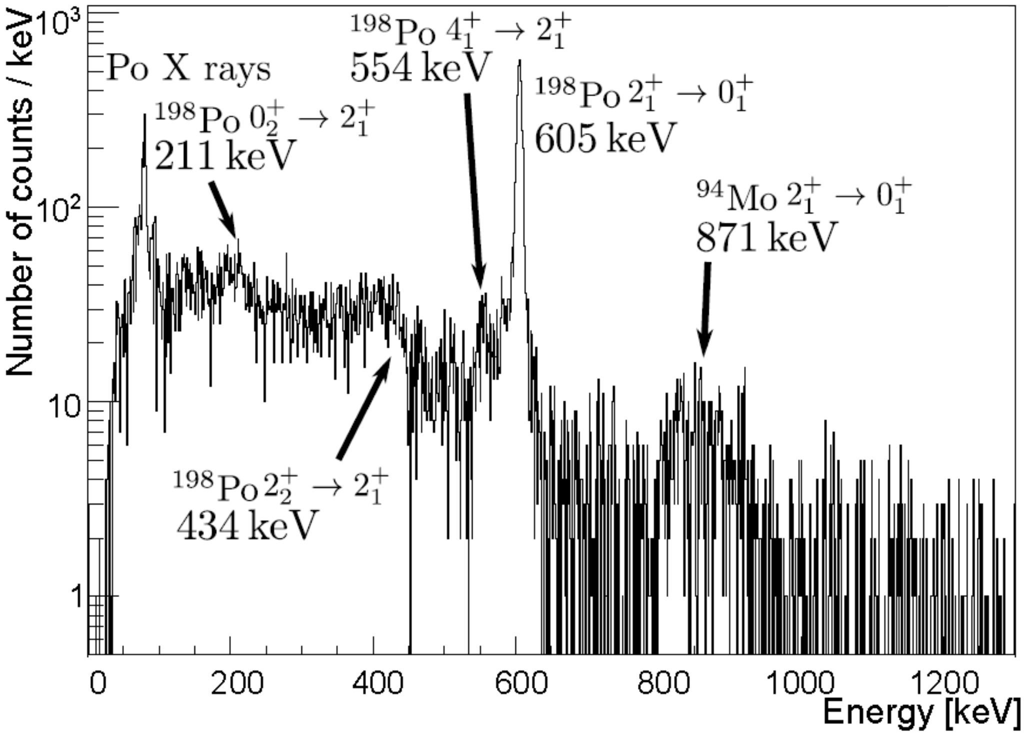


- χ^2 calculated for various combinations of $<2^+_1||E2||2^+_1>$ and $<2^+_1||E2||0^+_1>$ for the beam;
- solution corresponds to the minimum of the total χ^2_{total} for both beam and target nuclei;
- the 1σ uncertainty countour defined as the region of the surface for which $\chi^2 < \chi^2_{total,min} + 1$

- if more than two matrix elements involved – almost impossible to estimate errors !

Multistep Coulomb excitation of exotic even-even nuclei

^{198}Po @ 2.85 MeV/A on ^{94}Mo target, REX-ISOLDE



Nele Kesteloot, PhD thesis, KU Leuven, 2014
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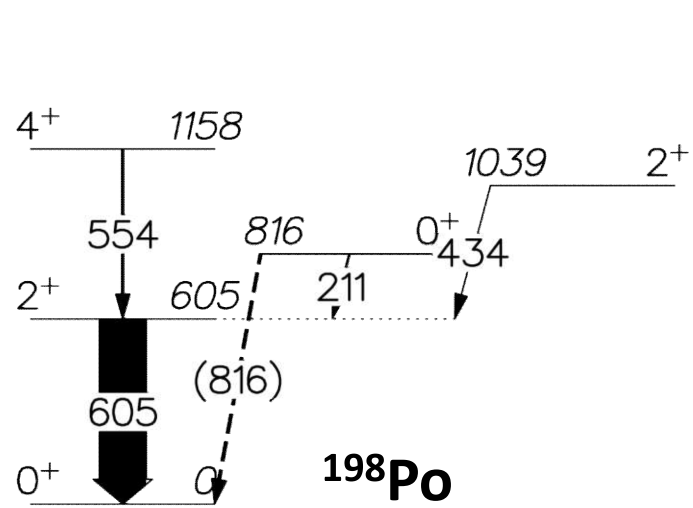


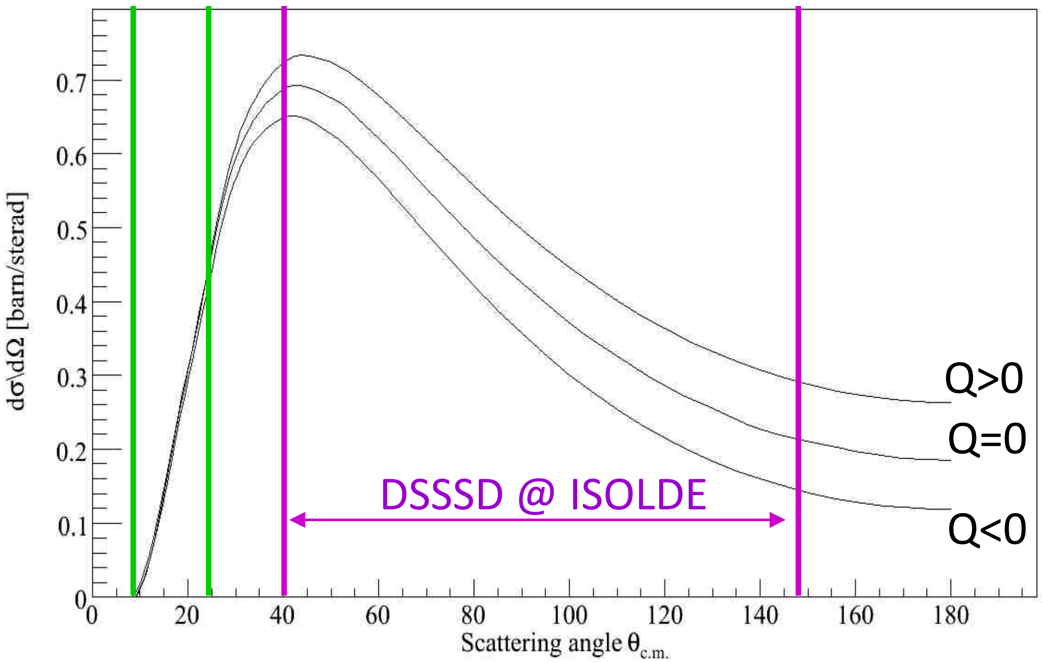
Case:

1. No complementary spectroscopic data.
2. More than two matrix elements involved.
3. Normalisation to the target excitation.

Problems:

1. Error calculations including correlations between MEs.
2. How to include contribution from uncertainty originating from the target excitation ?

Possible solutions



1. Normalisation to the *B(E2)* extracted from data sets where **no correlations** are observed

   lowest angular range → influence of quadrupole moment negligible → determination of the $B(E2;\, 2^+_1 \rightarrow 0^+_1)$

   [44]Ar: **M. Zielińska et al., PRC 80, 014317 (2009)**

2. Multistep Coulomb excitation and normalization to the target excitation.

GOSIA 2 → $B(E2;\, 2^+_1 \rightarrow 0^+_1)$ → contain information on uncertainty originating from the target excitation

\+

standard GOSIA → error calculation (including correlations) of *ME*s coupling higher - lying states

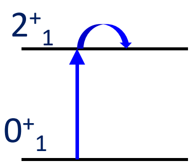
[196,198]Po: **N. Kesteloot, PhD thesis, KU Leuven, 2015**
PRC 92, 054301 (2015)

[182-188]Hg: **N. Bree, PhD thesis, KU Leuven, 2014**

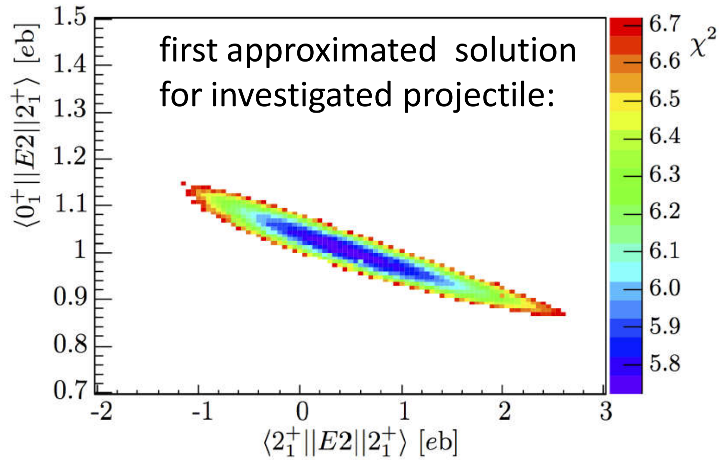
K. Wrzosek-Lipska et al, Eur. Phys. J. A (2019) 55: 130

GOSIA2⟺standard GOSIA analysis

GOSIA2; first approximation for projectile



projectile simplified level scheme



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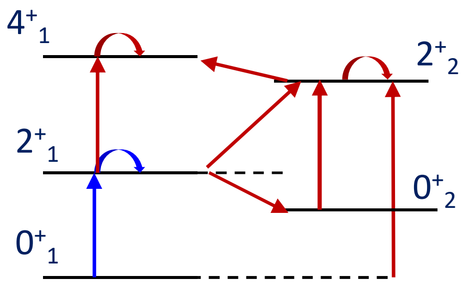
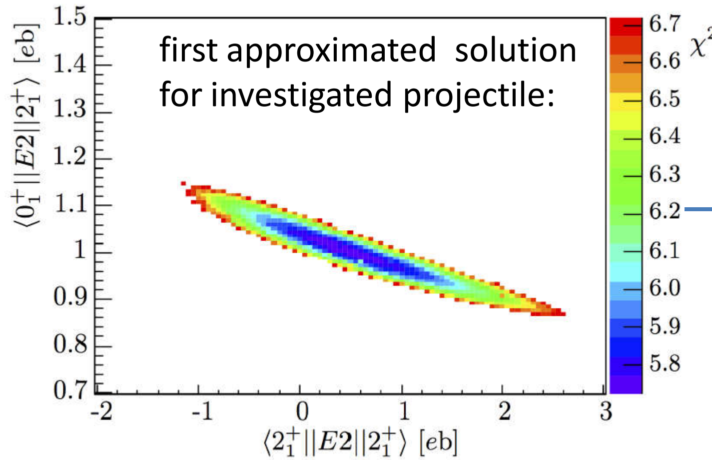


projectile full level scheme



$\langle 0_1^+ \| E2 \| 2_1^+ \rangle$

(serves as an absolute norm.)

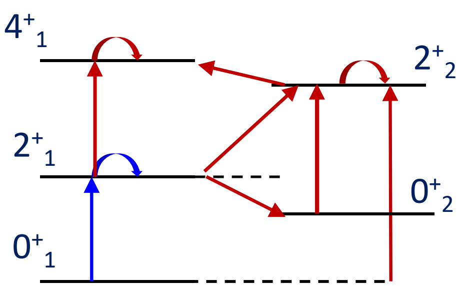
standard GOSIA; target excitation

normalisation constants C_{ij}

standard GOSIA; full minimisation for the projectile

GOSIA2 ⟺ standard GOSIA analysis

GOSIA2; first approximation for projectile



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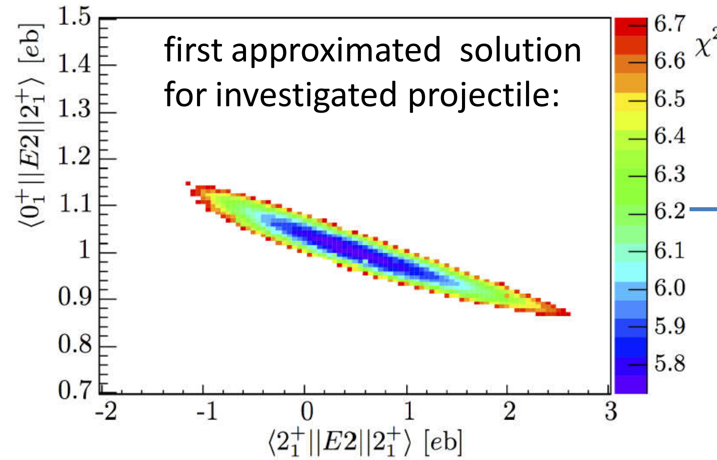

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standard GOSIA; target excitation

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standard GOSIA; full minimisation for the projectile

best fit matrix elements

GOSIA2; all matrix elements fixed, only **$<0^+_1||E2||2^+_1>$** and **$<2^+_1||E2||2^+_1>$ free**

$\langle 0_1^+ || E2 || 2_1^+ \rangle$

$\langle 2_1^+ || E2 || 2_1^+ \rangle$

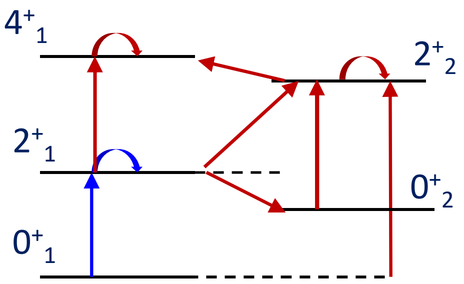
converged ?

NO

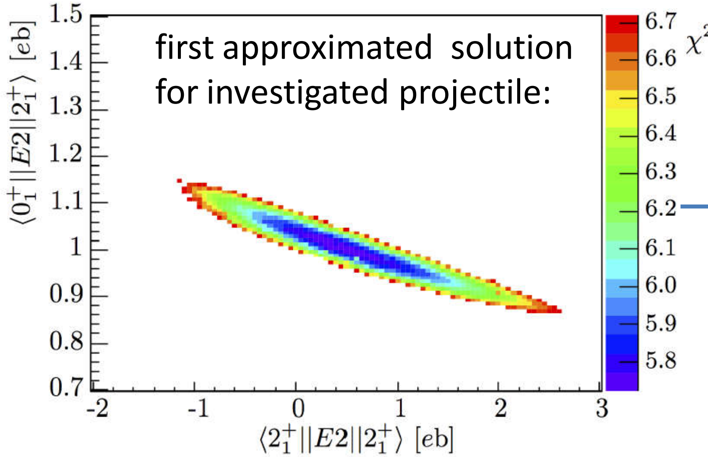
$\langle 0_1^+ || E2 || 2_1^+ \rangle$

GOSIA2 ⟺ standard GOSIA analysis

GOSIA2; first approximation for projectile



projectile full level scheme



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$\langle 0^+_1\|E2\|2^+_1\rangle$

$\langle 2^+_1\|E2\|2^+_1\rangle$

converged ?

NO

$\langle 0^+_1\|E2\|2^+_1\rangle$

YES

final solution !

Summary:

1. Normalisation constants required to convert **measured** γ-ray intensities to **absolute** Coulomb-excitation cross sections.
2. It is **not possible to impose an absolute normalisation** in the standard GOSIA. Only relative normalisation of one experiment to another can be imposed.
3. Normalization constants in Gosia analysis can be either **specified by user** or **fitted independently.**
4. Independent normalization (INR)
   a) known τ or B(E2, $2^+_1 \rightarrow 0^+_1$) determined from the lowest ϑ^{CM}
   b) number of experimental data large enough

   } standard Gosia
5. only one state populated + no life time information
   → normalization to the target excitation required

   } Gosia2
6. Multistep Coulomb excitation (no additional data available) – combined standard Gosia ↔ Gosia2 analysis:

   → final error bars of fitted matrix elements contain also uncertainty originating from the target excitation.

For more details and additional information…

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Special Article – Tools for Experiment and Theory

Analysis methods of safe Coulomb-excitation experiments with radioactive ion beams using the GOSIA code

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