Model assumptions in Coulomb-excitation analysis and other GOSIA tricks

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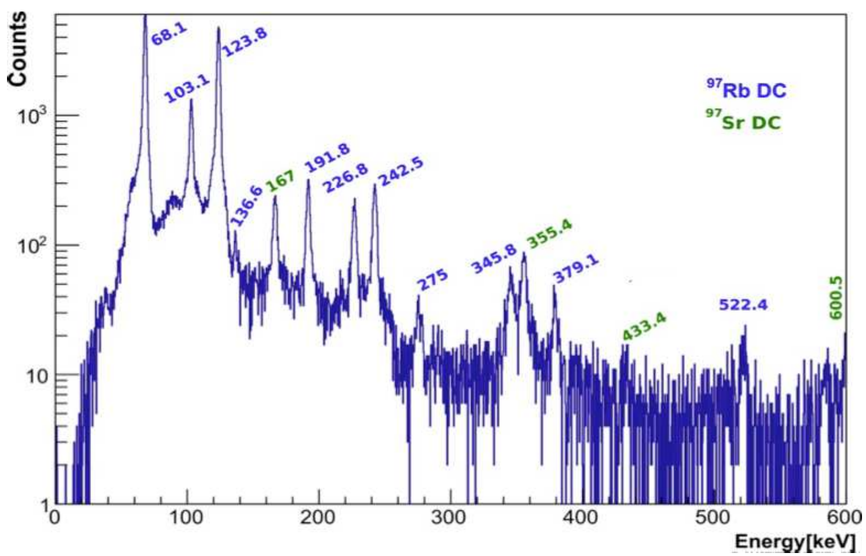
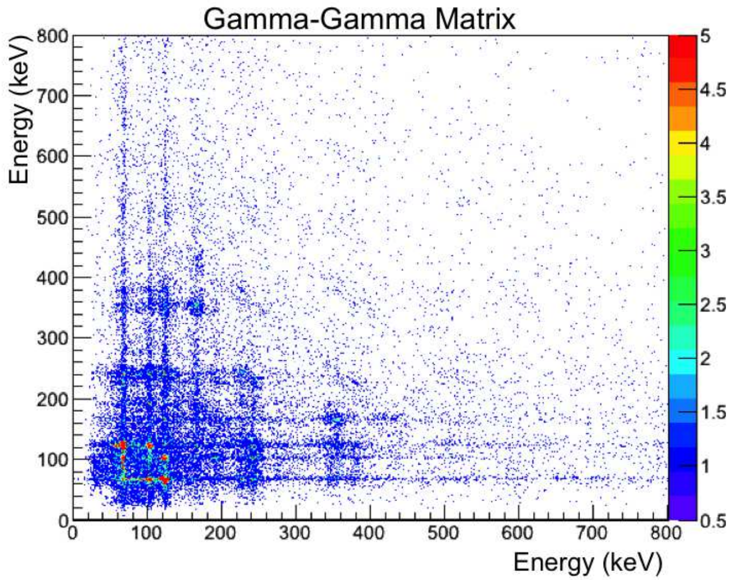
- Model assumptions for a well-deformed odd-A case: ^{97,99}Rb
- Efficiency of particle detectors
- Optimal subdivision of data
- Buffer states and sensitivity to unobserved transitions

Low-Energy Coulomb Excitation and Nuclear Deformation, chapter in: *The Euroschool on Exotic Beams - Vol.6*, Lecture Notes in Physics 1005, 43 (2022).

Coulomb excitation of $^{97-99}$Rb at ISOLDE

- identification of rotational bands in $^{97-99}$Rb (first observation of collective states in these nuclei!)
- statistics sufficient for gamma-gamma coincidences – level schemes established



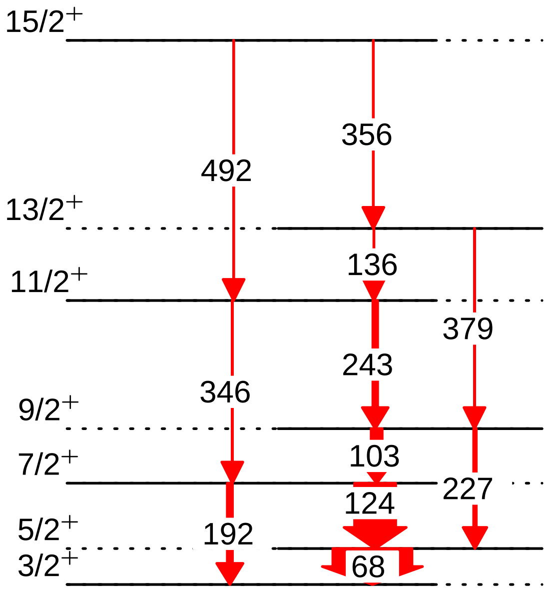


Ch. Sotty, Phys. Rev. Lett. 115 (2015) 172501

- Second step: extraction of E2 and M1 matrix elements using GOSIA code

Problems in Coulomb excitation data analysis (^{97}Rb)

- Cline's safe Coulomb excitation criterion not fulfilled for high CM angles
- efficiency for the 68 keV line uncertain
- 355 keV transition obscured by a line in ^{97}Sr
- underdetermined problem: 20 gamma rays, 24 matrix elements (E2 and M1)
- very strong correlations between matrix elements



Problems in Coulomb excitation data analysis (^{97}Rb) and solutions

- Cline's safe Coulomb excitation criterion not fulfilled for high CM angles

→ 15 % of statistics excluded from the analysis

- efficiency for the 68 keV line uncertain

→ would be a natural choice for normalisation but had to be excluded from the analysis

- 355 keV transition obscured by a line in ^{97}Sr

→ intensity obtained from gamma-gamma coincidences

- underdetermined problem: 20 gamma rays, 24 matrix elements (E2 and M1)

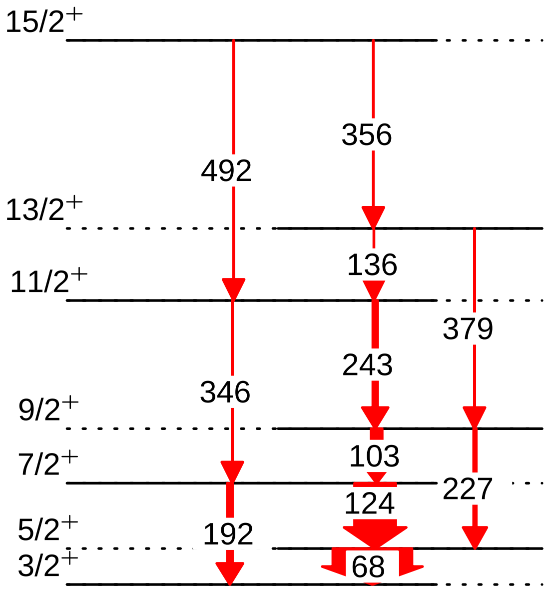
→ model assumptions necessary: Alaga rules

$$\langle KI_f||E2||KI_i\rangle = \sqrt{(2I_i+1)}(I_i,K,2,0|I_f,K)\sqrt{\frac{5}{16\pi}}eQ_0$$

⇒ within rotational model E2 branching ratio depends on spins only (Q_0 cancel out)

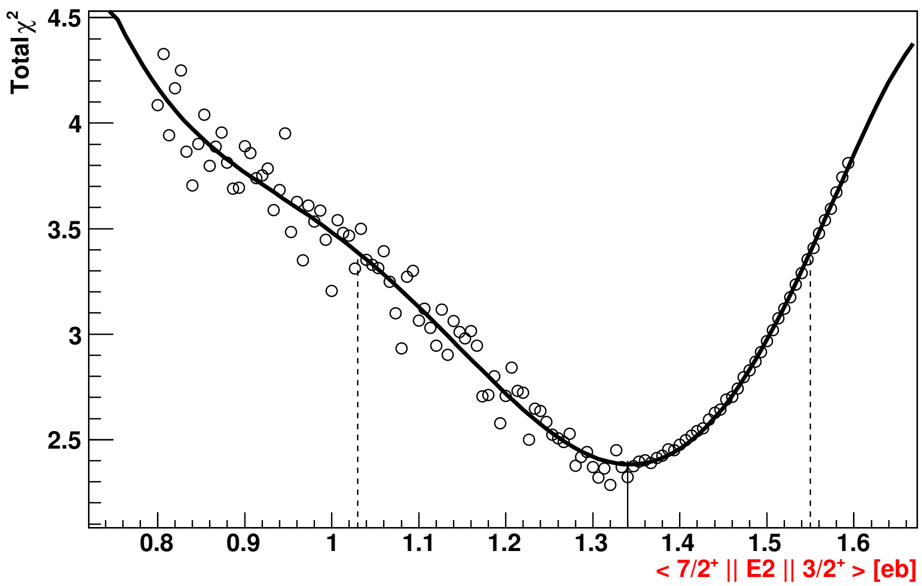
- very strong correlations between matrix elements

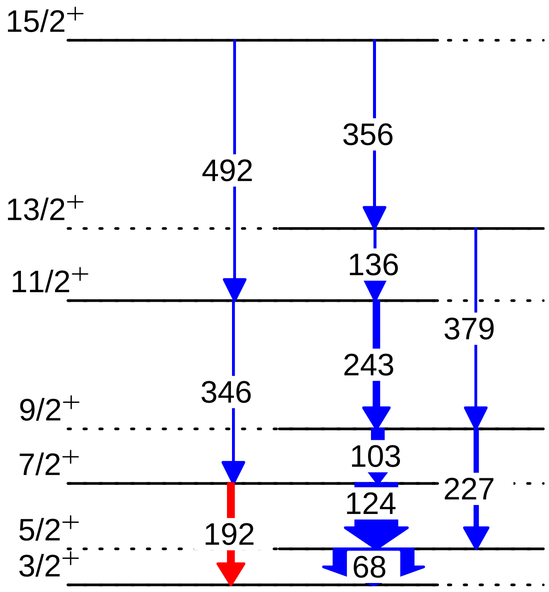
→ large uncertainties for low-lying transitions



Normalisation to target excitation

- for each value of $\langle 7/2^+ \| E2 \| 3/2^+ \rangle$ all remaining matrix elements in Rb and Ni are fitted to observed gamma-ray intensities and known spectroscopic data (GOSIA2)
- Alaga rules assumed for each pair of I $\rightarrow$ I-1 and I $\rightarrow$ I-2 E2 transitions



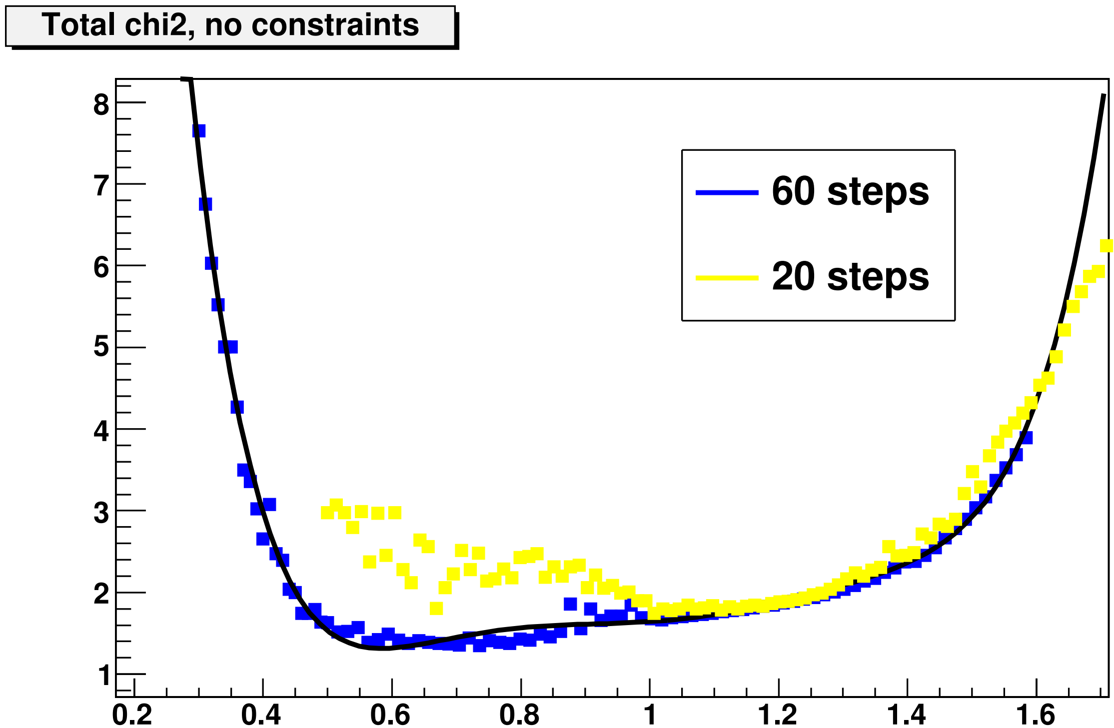


- for all other transitions a standard GOSIA1 analysis assuming this value of $\langle 7/2^+ \| E2 \| 3/2^+ \rangle$

Ch. Sotty, Phys. Rev. Lett. 115 (2015) 172501

Normalisation to target excitation

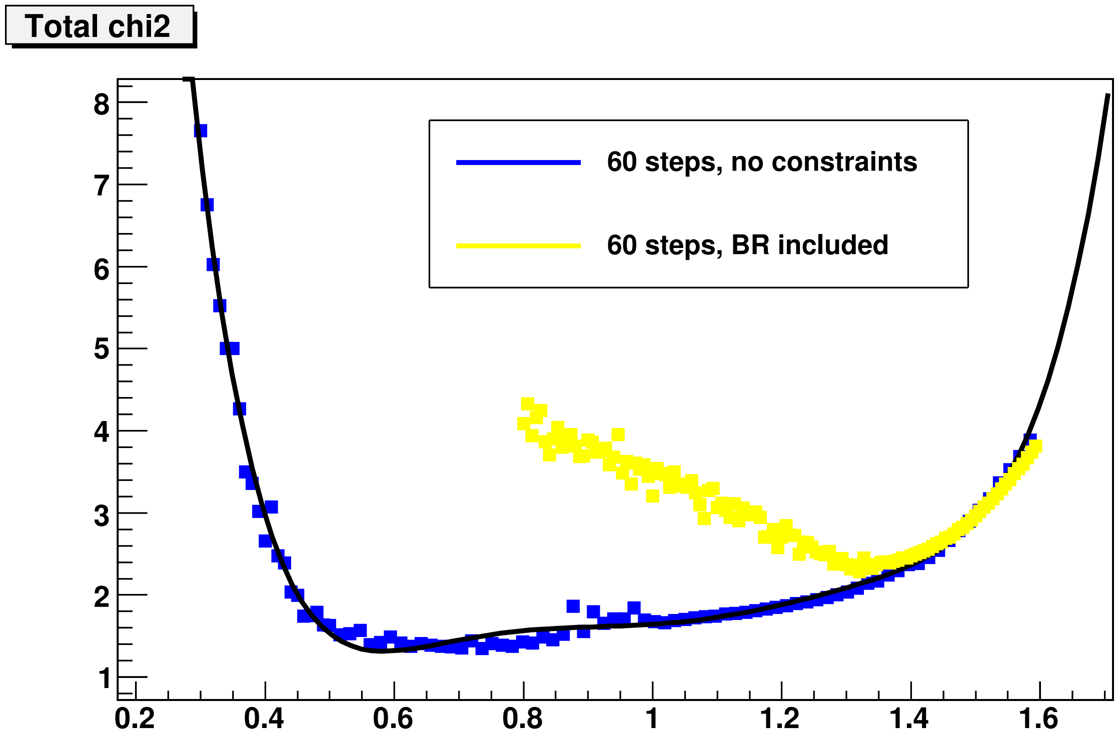
Convergence problems



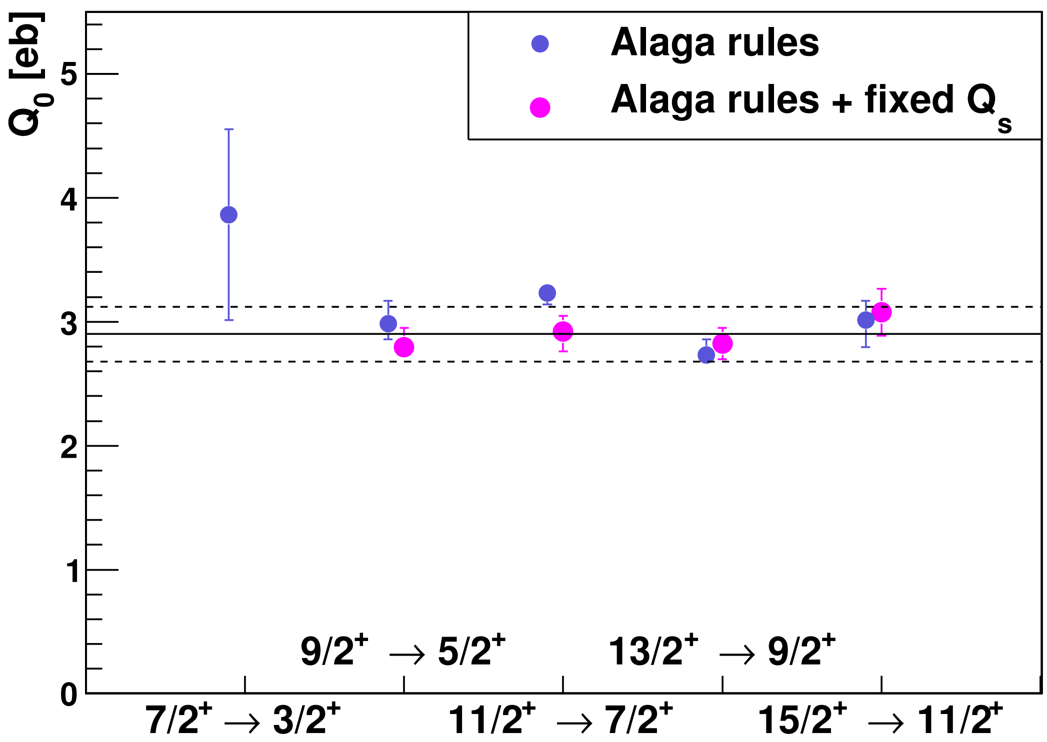
- fluctuations due to a local χ^2 minimum, more iterations give a more smooth dependence (and a new global minimum)
- smooth parts of the χ^2 curve don't change much

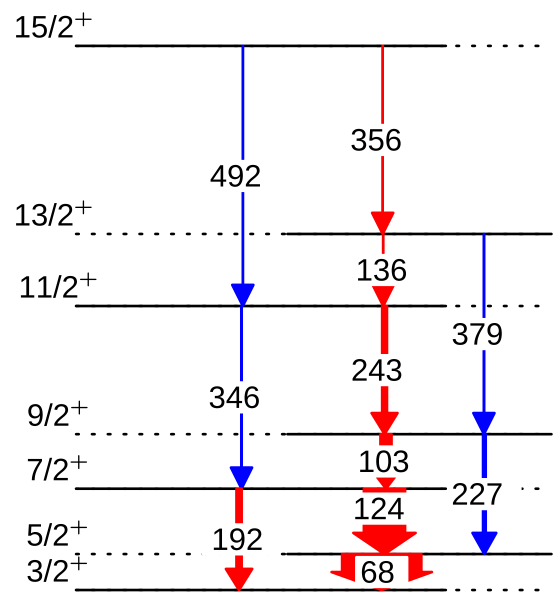
Normalisation to target excitation

Different minimum if E2 branching ratios imposed



Results: deformation of ^{97}Rb



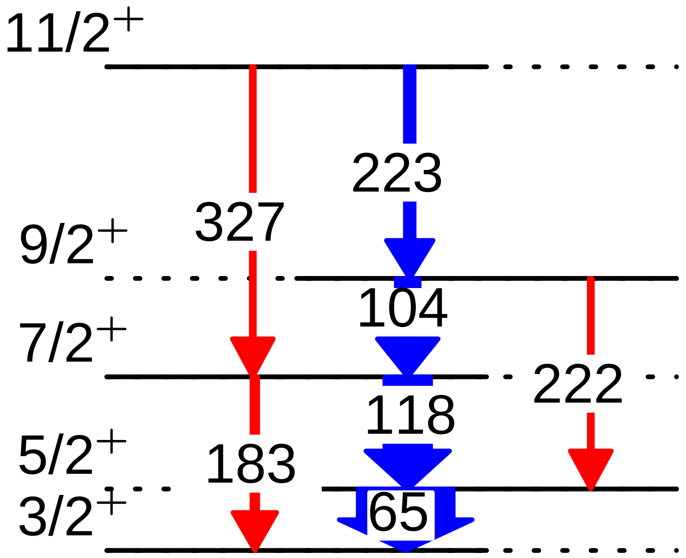


- two different assumptions give consistent results for 4 matrix elements
- these 4 transitions are populated in multi-step excitation → matrix elements basically given by the observed intensity ratios in ^{97}Rb (weak dependence on adopted normalisation)
- results consistent with the ground-state quadrupole moment measured in laser spectroscopy (horizontal lines)

Next step: ^{99}Rb

Problems we know already from ^{97}Rb:

- Cline's safe Coulomb excitation criterion not fulfilled for high CM angles
- efficiency for the 65 keV line uncertain
- very strong correlations between matrix elements

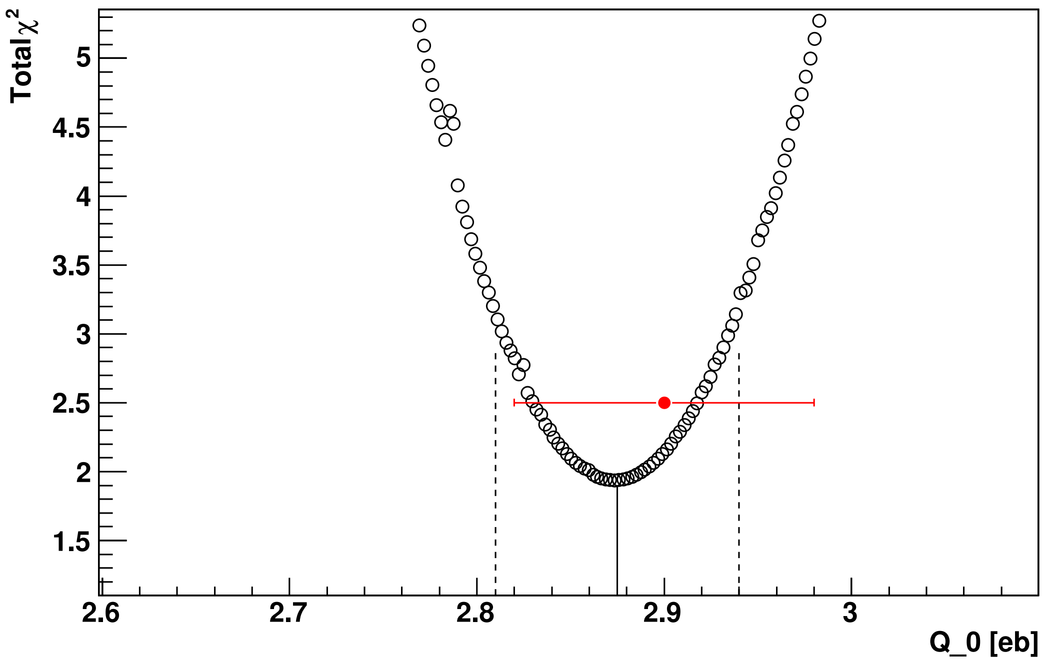


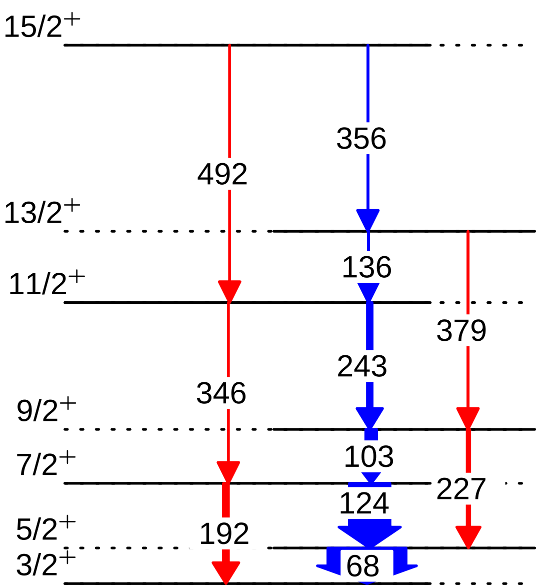
New problems:

- very low statistics (few hundred counts in the strongest line)
- target excitation not observed
- unresolved doublet at 222 keV
- extremely underdetermined problem: 6 gamma rays, 15 matrix elements)

^{99}Rb: proposed solution and test on ^{97}Rb data

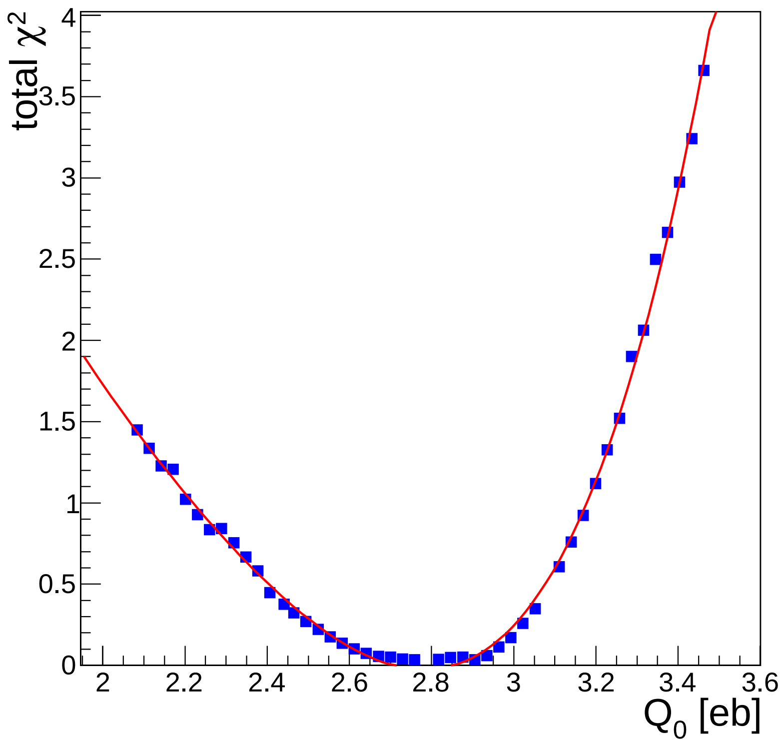
- matrix elements in the upper part of a strongly deformed rotational band related to observed intensity ratios in the nucleus under study (no external normalisation required)
- all E2 matrix elements (including Q_s) coupled using rotational model
- then we fit only M1 matrix elements and one Q_0 to measured gamma-ray intensities
- tested on ^{97}Rb data, result consistent with weighted average of Q_0 values obtained in standard analysis

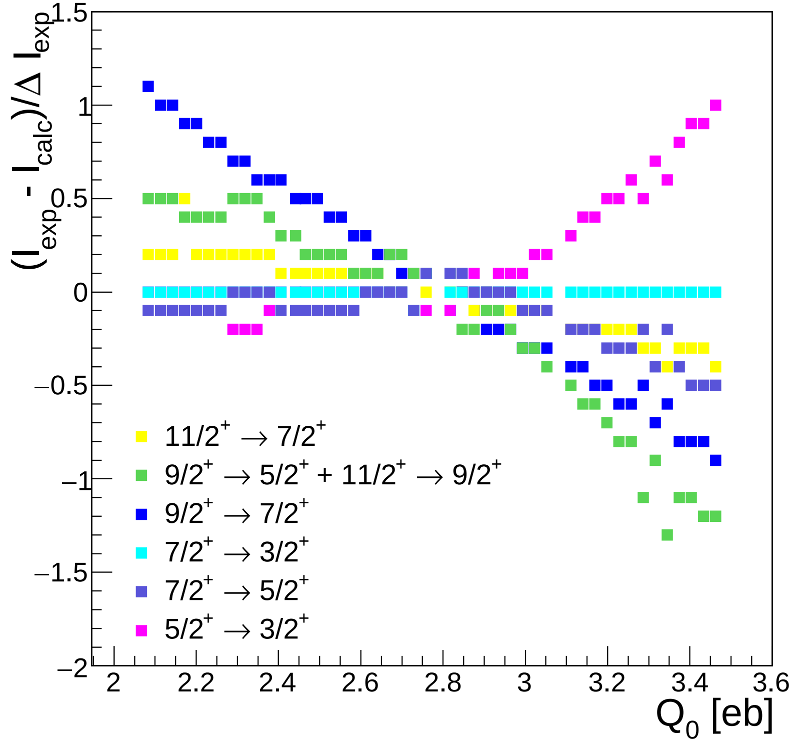


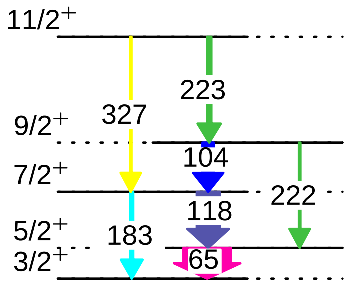


^{99}Rb: results

- 4 M1 matrix elements and one Q_0 fitted to measured gamma-ray intensities in ^{99}Rb

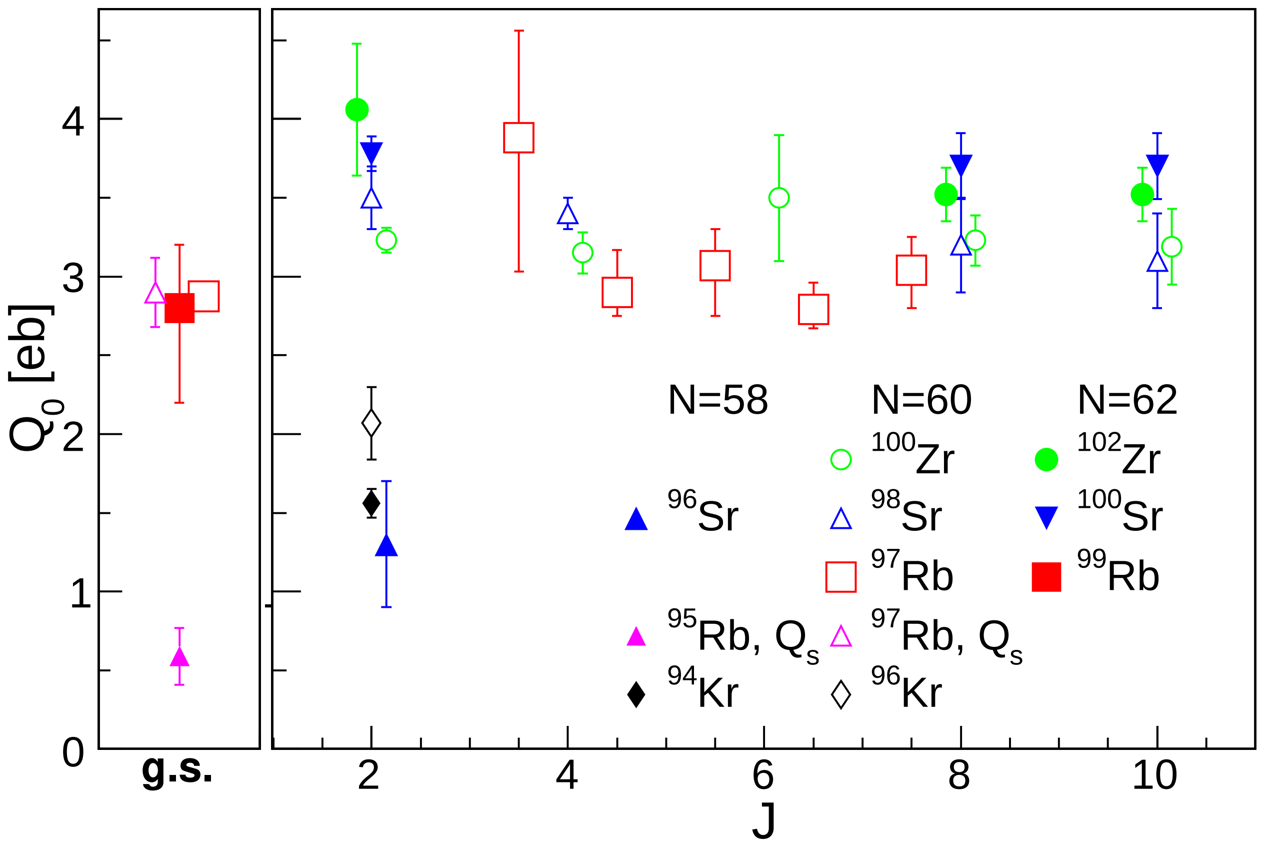






- one clear χ^2 minimum for all observed transitions
- precision rather low due to limited statistics

Deformation of ^{99}Rb: comparison with ^{97}Rb

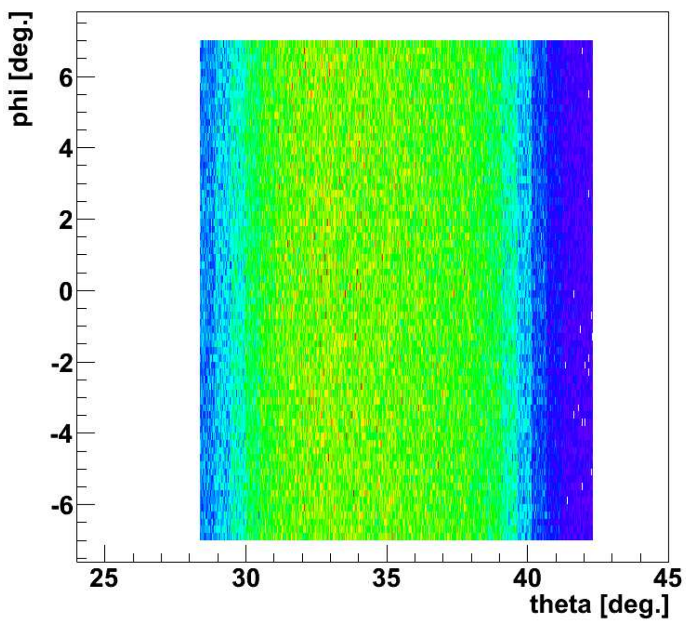


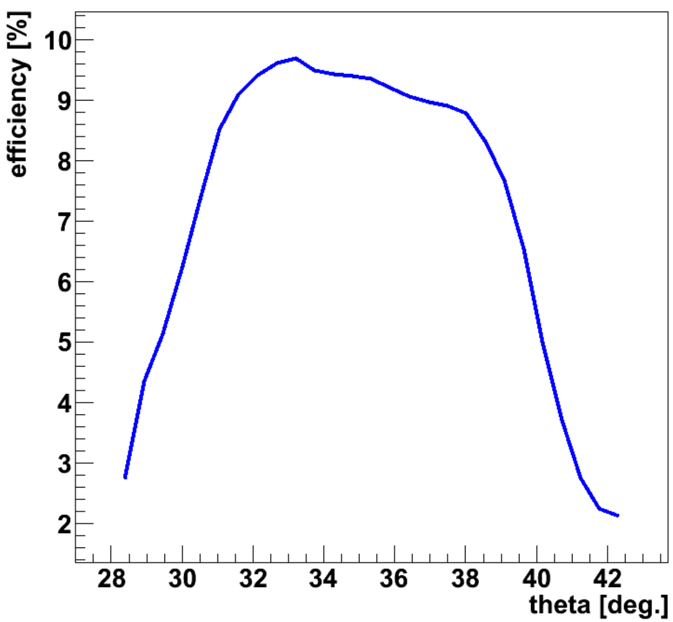
Ch. Sotty, Phys. Rev. Lett. 115 (2015) 172501

Efficiency of particle detectors

two ways to account for it

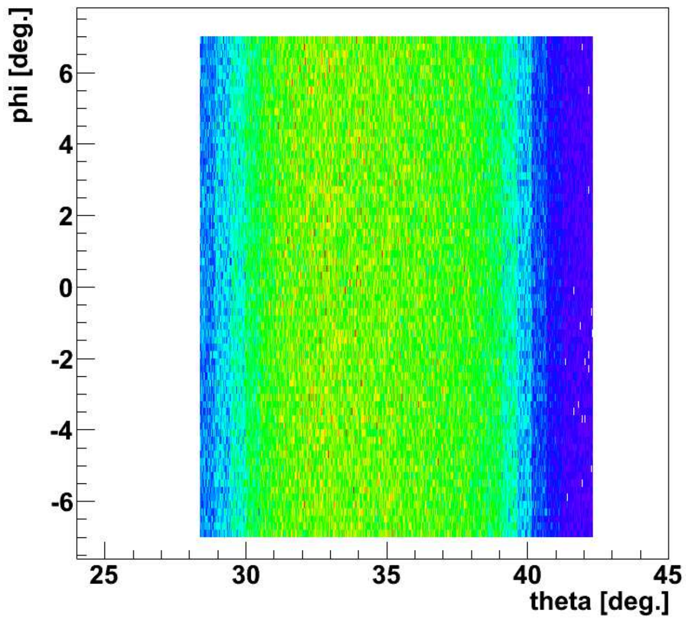
Efficiency of a particle detector

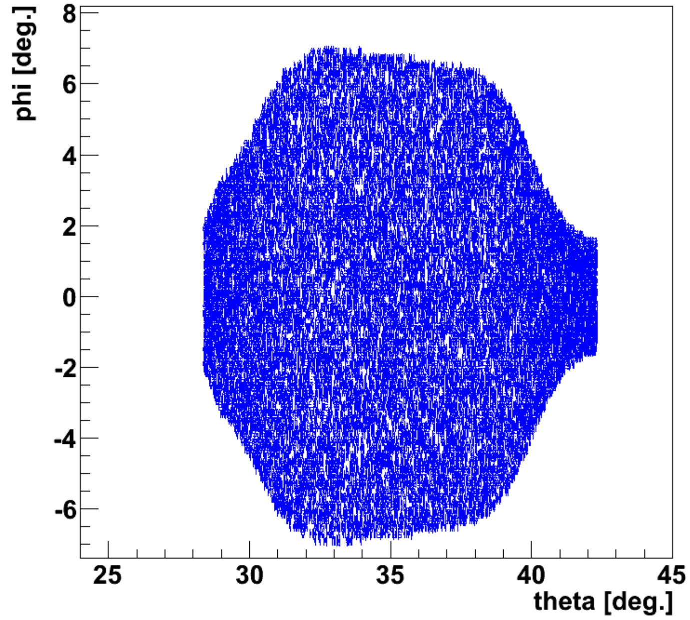




- efficiency doesn't have to be uniform for all the detector's area
- some parts of the detectors can stop working at some point
- if the detector has a uniform efficiency lower than 100% no need to account for it in GOSIA (this effect will be naturally included in the normalisation constant of the experiment)

If the efficiency changes as a function of θ...

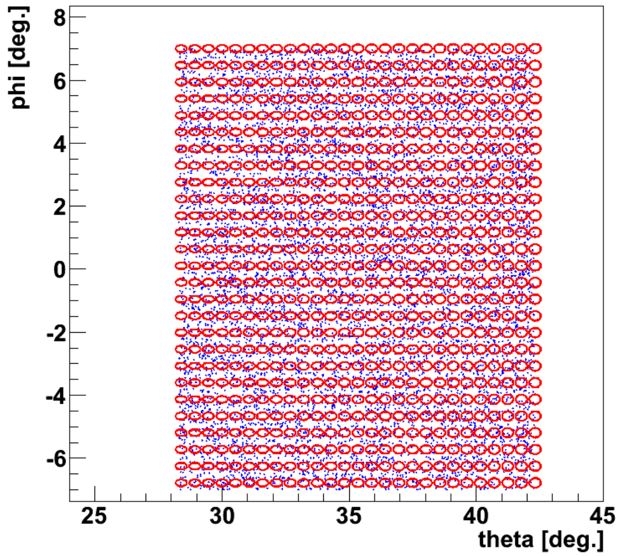




Solution 1:

- Coulomb-excitation cross section depends on θ scattering angle – one can modify detector shape in φ to take the efficiency into account
- applicable if we have a symmetric gamma detection set-up (gamma-particle correlations smoothed out)
- φ range covered by the detector scaled according to efficiency: true range (here: (-7°,7°)) where efficiency is maximal, reduced to (-3.5°,3.5°) where it's only 50% of the maximum value, etc.

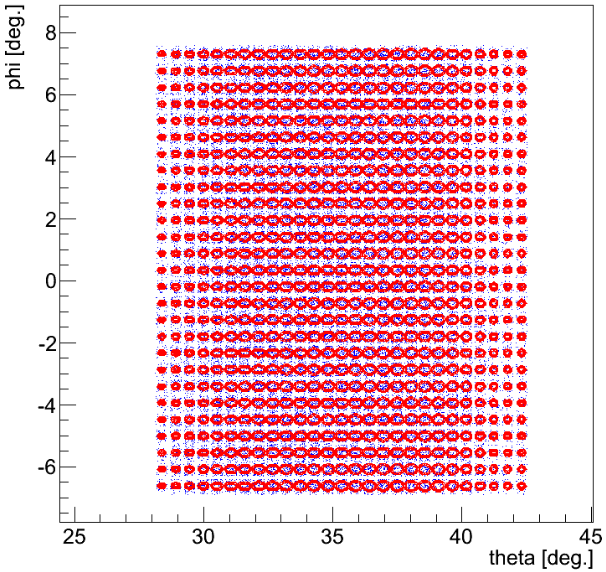
Efficiency of the particle detector: second solution



OP,INTG
3, 3, 683.2, 690.7, 28.38, -6.62, 0.223
683.2, 687, 690.7
3, 3, ...
...

- detector shape approximated by a large number (here: 729) of small circular detectors
- first step: comparison with a 100% efficiency detector, angular range covered by a single detector chosen to reproduce the Rutherford cross section

Efficiency of the particle detector: second solution



OP,INTG
3, 3, 683.2, 690.7, 28.38, -6.62, 0.119
683.2, 687, 690.7
3, 3, ...
...

- detector shape approximated by a large number (here: 729) of small circular detectors
- first step: comparison with a 100% efficiency detector, angular range covered by a single detector chosen to reproduce the Rutherford cross section
- second step: detector areas scaled according to efficiency (θ_{half} scaled as $\sqrt{A}$)
- does not change the φ coverage of the detector – better if particle-gamma correlations important

Comparison of results

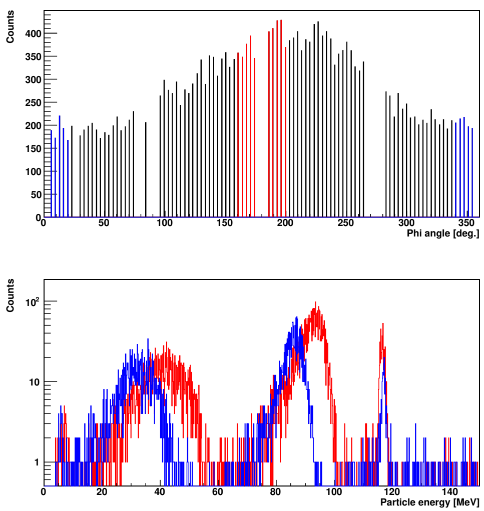
Integrated yields, normalised to $2_1^+ \to 0_1^+$ (YCOR)

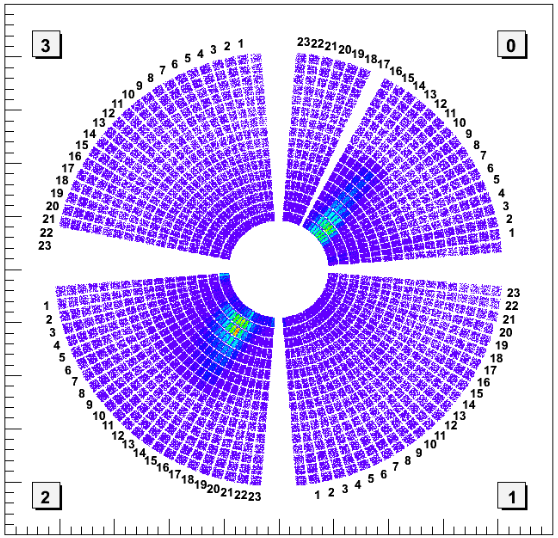
transition	eff vs standard	PIN vs standard	PIN, eff vs PIN
$14_1^+ \to 12_1^+$	18.5%	-2.8%	19.3%
$12_1^+ \to 10_1^+$	13.6%	-2.0%	13.6%
$10_1^+ \to 8_1^+$	8.2%	-1.0%	7.8%
$6_2^+ \to 6_1^+$	4.3%	-0.4%	3.5%
$8_1^+ \to 6_1^+$	0.7%	0.0%	0.0%
$4_2^+ \to 4_1^+$	-3.4%	0.7%	-4.0%
$2_2^+ \to 2_1^+$	-1.2%	0.0%	-1.2%
$6_1^+ \to 4_1^+$	-1.7%	0.7%	-2.7%
$4_1^+ \to 2_1^+$	-1.1%	0.5%	-1.6%
$2_1^+ \to 0_1^+$	0.0%	0.0%	0.0%
Rutherford	-23.2%	0.5%	-25.7%

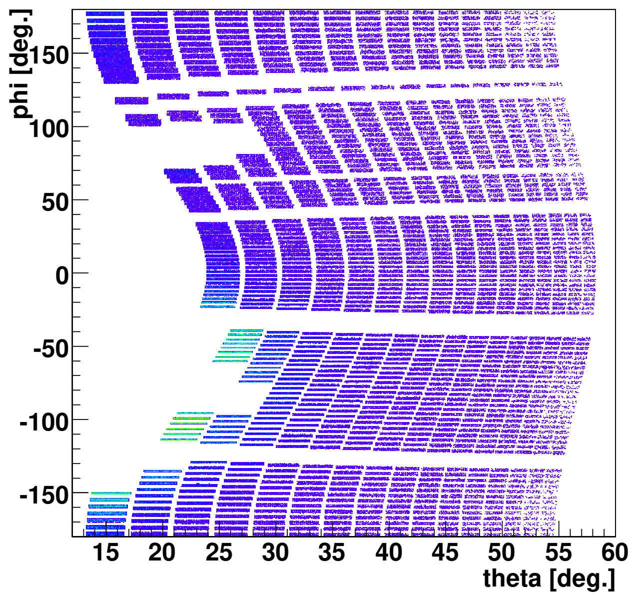
- both solutions work reasonably well
- corrections important for multi-step and non-yrast states

Complicated detector shapes: example of a ^{44}Ar study from GANIL

MZ et al, Phys. Rev. C 80, 014317 (2009)





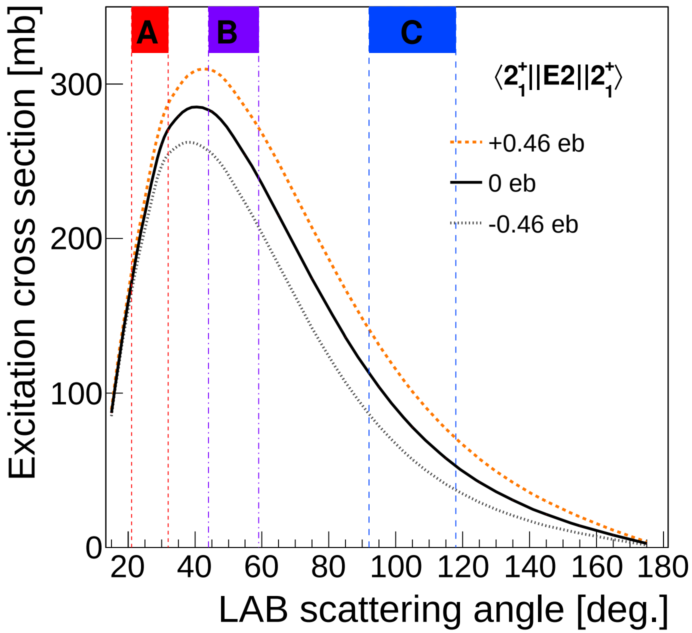


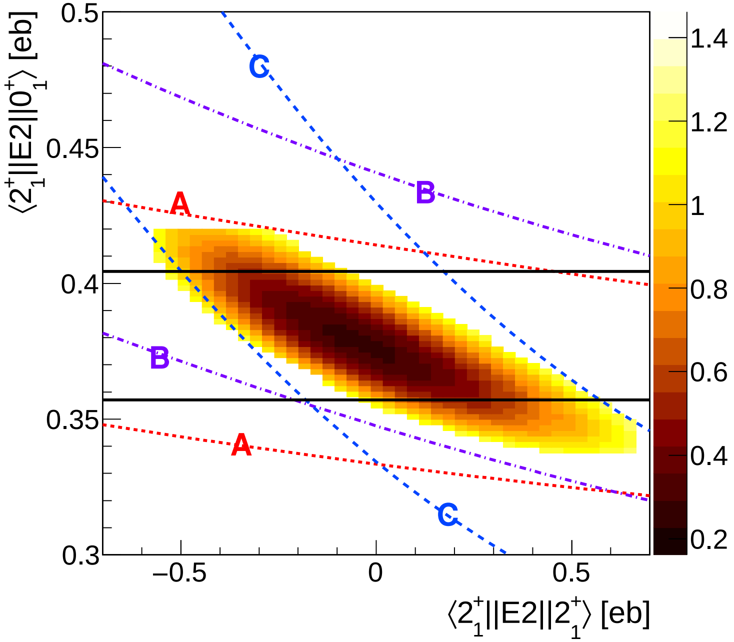
- missing pixels and displacement of the detector with respect to the beam spot
- complicated shape succesfully approximated by >1400 circular detectors
- influence of the approximation on calculated gamma-ray yields estimated to be below 4% (included in the uncertainties)
- compared to no corrections for detector shape, effect $\geq$ 15% for inner rings

Optimal subdivision of Coulomb-excitation data

Where is sensitivity to quadrupole moments coming from?

^{76}Zn, HIE-ISOLDE data from: A. Illana, MZ *et al.*, submitted to PRC





- compromise between number of subdivisions and statistics
- useful to have a range where the influence of $\langle 2^+||E2||2^+\rangle$ is negligible (horizontal cut), but not always possible
- for high CM angles influence of quadrupole moment should be higher than statistical error of the gamma yield
- if two cuts in $\langle 2^+||E2||2^+\rangle$, $\langle 2^+||E2||0^+\rangle$ plane are really close, probably you will gain more by combining the statistics

Effect of unobserved transitions

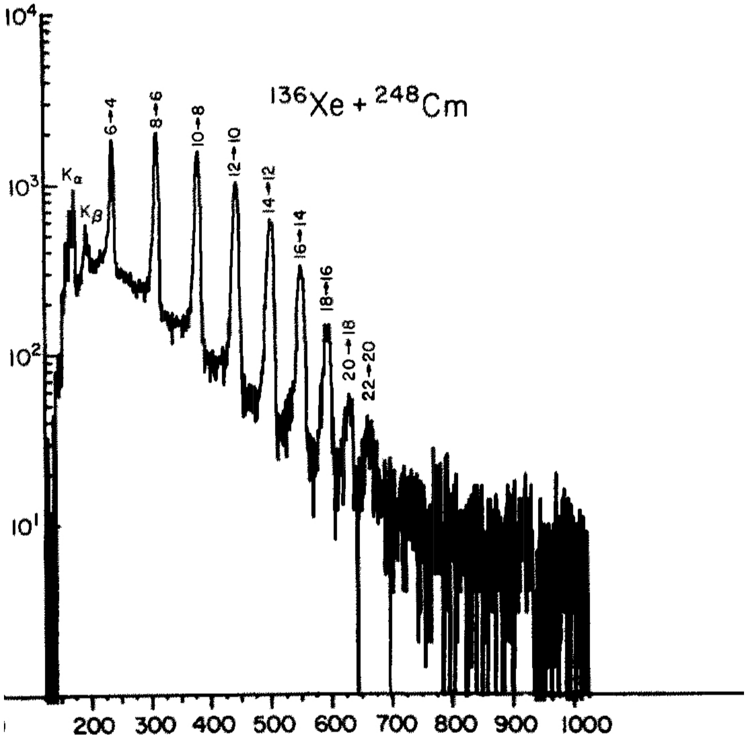
Buffer states

- reorientation effect can be comparable with population of higher-lying states
- when analysing Coulomb-excitation data, we should include buffer states on top of bands to account for possible excitation of higher-lying states
- otherwise we get incorrect quadrupole moments, or, more rarely, even incorrect in-band B(E2) values between the higher-lying states
- rotational model can be used to estimate starting values of ME
- one buffer state on top of a band should be enough, as demonstrated by the next example

Buffer states

T. Czosnyka et al, Nucl. Phys. A458 (1986) 123

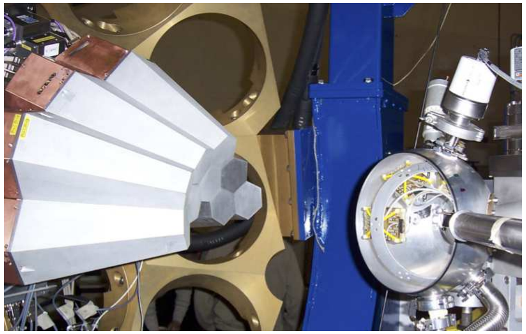
- ^{248}Cm Coulomb-excited with a ^{136}Xe beam, observation of states up to 22^+
- very collective ground-state band, no other states observed

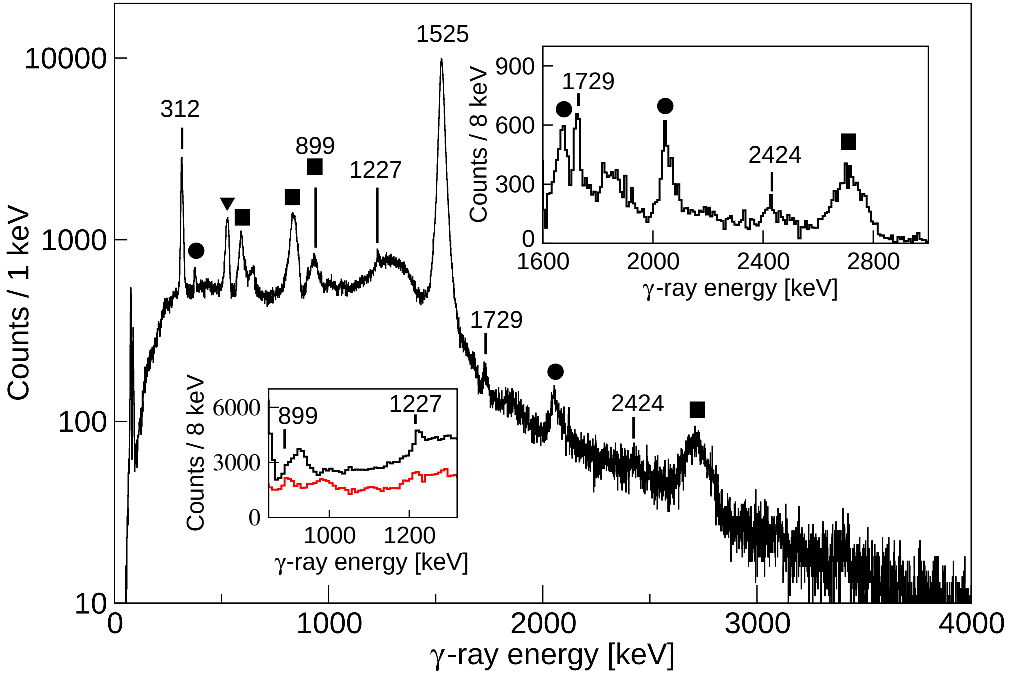


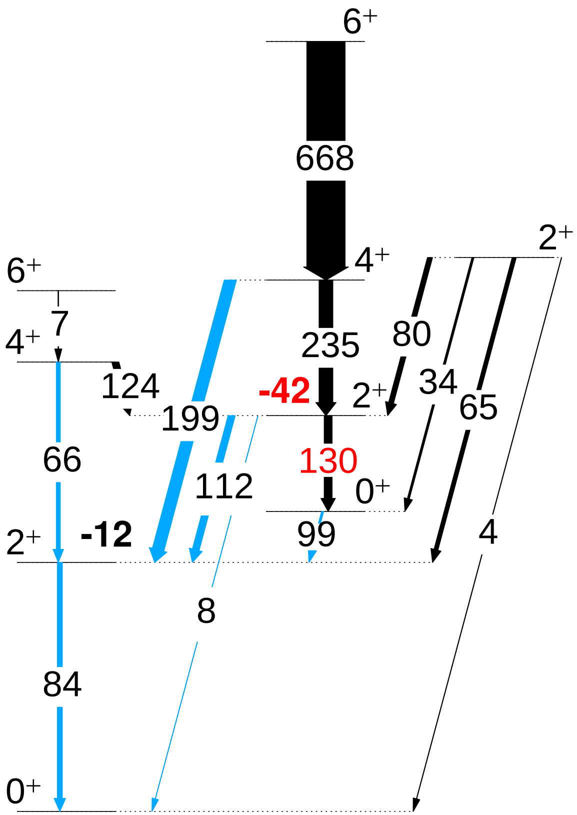
transition	levels up to 30^+	levels up to 22^+	up to 22^+, no $Q_s(22^+)$
$24^+ \rightarrow 22^+$	2.4 mb	–	–
$22^+ \rightarrow 20^+$	9.3 mb	9.6 mb (+3%)	11.1 mb (+20%)
$20^+ \rightarrow 18^+$	29.0 mb	28.8 mb (-<1%)	28.0 mb (-3.5%)
$18^+ \rightarrow 16^+$	73.0 mb	73.0 mb (0%)	73.3 (+<1%)

Coulomb excitation of ^{42}Ca at LNL

- Targets: ^{208}Pb, ^{197}Au, 1mg/cm^2
- AGATA: 3 triple clusters
- DANTE: 3 MCP detectors, θ range: 100-144°





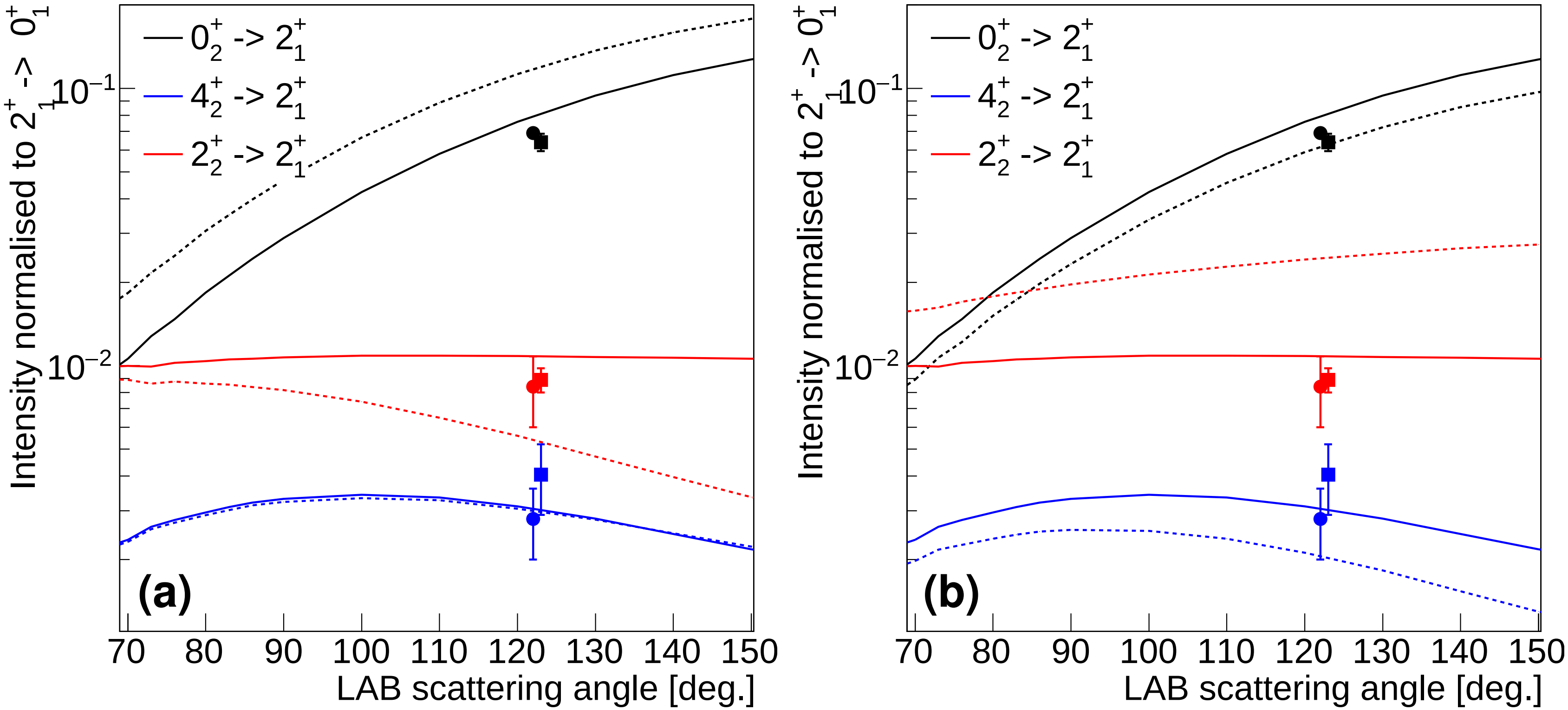


- first population of a superdeformed band in Coulomb excitation
- measured quadrupole moment of 2_2^+ corresponds to $\beta = 0.48(14)$

K. Hadyńska-Klęk et al, PRL 117 (2016) 062501

Sensitivity to matrix elements corresponding to an unobserved transition

MZ, K. Hadyńska-Klęk, EPJ Web Conf 178 (2018) 02014



dotted: $\langle 2_2^+ \| \mathrm{E2} \| 0_2^+ \rangle = 0$ dotted: $\langle 2_2^+ \| \mathrm{E2} \| 2_2^+ \rangle = 0$

- opposite effects of $\langle 2_2^+ \| \mathrm{E2} \| 0_2^+ \rangle$ and $\langle 2_2^+ \| \mathrm{E2} \| 2_2^+ \rangle$ on the population of 0_2^+ and 2_2^+ states
- population of the 4_2^+ state sensitive only to $\langle 2_2^+ \| \mathrm{E2} \| 2_2^+ \rangle$