

$B^0 \rightarrow 3K$ decays in an unitary approach and $B \rightarrow D^*(D\pi)l\nu$ decays

Project:

*"Phenomenological analyses of high precision B and D decay
data and tests of the Standard Model"*

Emi Kou: *Laboratoire de l'Accélérateur Linéaire (LAL)
Centre Scientifique d'Orsay*

Robert Kamiński: *Division of Theoretical Physics
Institute of Nuclear Physics PAS, Kraków*

IN2P3-COPIN meeting, Warszawa 20 XI 2023

So far, scientific achievements of our cooperation for B and D decays:

For period 2007-2022:

- 15 papers (1/year),
- 264 citations (17.6/paper),
- 9 proceedings: MESON, Few-Body Problems, ICHEP, Hadron, EPS-HEP, Charm, Helmholtz Summer School
- 6 articles in: 5 in Phys. Rev. D and 1 in Acta Physica Polonica B,

For these 6 articles:

- 259 citations (43/paper)
- Citations/paper/year = 2.9 (avg)
- Number of authors: 5.2 (avg)

Scientific achievements:

J. P. Dedonder, R. Kamiński, L. Leśniak and B. Loiseau,
"Dalitz plot studies of $D^0 \rightarrow K_S^0 K^+ K^-$ decays in a factorization approach",
Phys. Rev. D **103** (2021) no.11, 114028,
[7 citations](#), [31 pages](#)

D. Boito, J. P. Dedonder, B. El-Bennich, R. Escribano, R. Kaminski, L. Lesniak and B. Loiseau,
"Parameterizations of three-body hadronic B- and D-decay amplitudes in terms of analytic and unitary meson-meson form factors",
Phys. Rev. D **96** (2017) no.11, 113003
[34 citations](#), [38 pages](#)

J. P. Dedonder, R. Kaminski, L. Lesniak and B. Loiseau,
"Dalitz plot studies of $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays in a factorization approach",
Phys. Rev. D **89** (2014) no.9, 094018
[34 citations](#), [34 pages](#)

J. P. Dedonder, A. Furman, R. Kaminski, L. Lesniak and B. Loiseau,
"S-, P- and D-wave final state interactions and CP violation in $B^\pm \rightarrow \pi^\pm \pi^\mp \pi^\pm$ decays",
Acta Phys. Polon. B **42** (2011), 2013
[50 citations](#), [31 pages](#)

O. Leitner, J. P. Dedonder, B. Loiseau and R. Kaminski,
"K* resonance effects on direct CP violation in $B \rightarrow \pi \pi K$ ",
Phys. Rev. D **81** (2010), 094033 [erratum: Phys. Rev. D **82** (2010), 119906]
[24 citations](#), [22 pages](#)

B. El-Bennich, A. Furman, R. Kaminski, L. Lesniak, B. Loiseau and B. Moussallam,
"CP violation and kaon-pion interactions in $B \rightarrow K \pi^+ \pi^-$ decays",
Phys. Rev. D **79** (2009), 094005 [erratum: Phys. Rev. D **83** (2011), 039903]
[110 citations](#), [28 pages](#)

Scientific achievements:

O. Leitner, J. P. Dedonder, B. Loiseau and R. Kaminski,
“ K^* resonance effects on direct CP violation in $B \rightarrow \pi\pi K$ ”,
Phys. Rev. D **81** (2010), 094033 [erratum: Phys. Rev. D **82** (2010), 119906]
24 citations, 22 pages

Decays: $B^\pm \rightarrow \pi^+\pi^-K^\pm$ and $\bar{B}^0 \rightarrow \pi^+\pi^-\bar{K}^0$

Method:

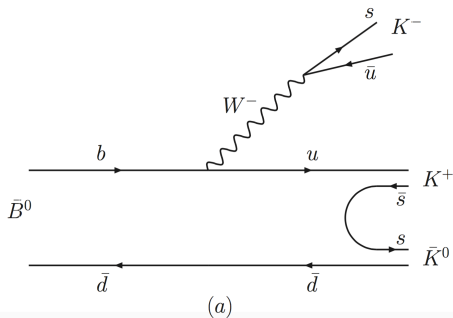
- QCD factorization schem (QCDF),
- Unitary final state interactions (FSI),
- The weak decay amplitudes: the tree and annihilation contributions.
- two complex parameters - fit to the effective mass and helicity angle distribution.

Results:

- $K_0^*(1430)$ branching ratios and direct CP violation asymmetries which compare well to those of Belle and BABAR Collaboration.

Factorization scheme

$$\mathcal{H}_1^{(p)} = \langle \bar{K}^0 K^+ | (\bar{u}b)_{V-A} | \bar{B}^0 \rangle \cdot \langle K^- | (\bar{s}u)_{V-A} | 0 \rangle \left[a_1 \delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p) r_\chi^K \right]$$



Scientific achievements:

O. Leitner, J. P. Dedonder, B. Loiseau and R. Kaminski,
“ K^* resonance effects on direct CP violation in $B \rightarrow \pi\pi K$ ”,
Phys. Rev. D **81** (2010), 094033 [erratum: Phys. Rev. D **82** (2010), 119906]
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B. El-Bennich, A. Furman, R. Kaminski, L. Lesniak, B. Loiseau and B. Moussallam, “*CP violation and kaon-pion interactions in $B \rightarrow K\pi^+\pi^-$ decays*”, Phys. Rev. D **79** (2009), 094005 [erratum: Phys. Rev. D **83** (2011), 039903]
106 citations, 28 pages

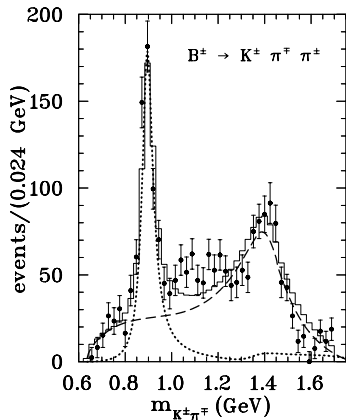
Method:

- The weak amplitudes: QCDF and penguin amplitudes,
- fitted to the $K\pi$ effective mass and helicity angle distributions (Belle and BaBar),
- the $K\pi$ scalar and vector form factors: from Muskhelishvili-Omnès coupled channel equations with chiral symmetry and asymptotic QCD constraints,

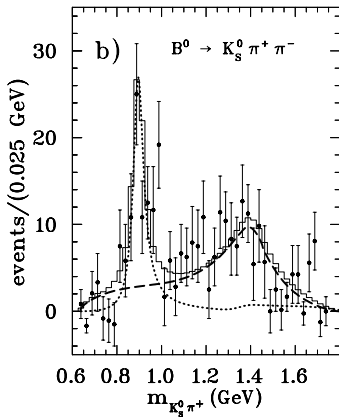
Results:

- \mathcal{B} for $B \rightarrow K^*(892)\pi$ and $B^\pm \rightarrow K_0^*(1430)\pi^\pm$, $K_0^*(1430) \rightarrow K^\pm\pi^\mp$ and ACP
- reduction of LARGE SYSTEMATIC UNCERTAINTIES in \mathcal{B}^{exp} for $B \rightarrow K_0^*(1430)\pi$

Examples of our fits:



BABAR



Belle

Table: Branching fractions for the $B \rightarrow K^*(892)\pi$ decays

Decay mode	\mathcal{B}^{exp}	$\mathcal{B}^{\text{exp}}(0.82, 0.97)$	model	model [$c_i^D \equiv 0$]
$B^- \rightarrow [\bar{K}^{*0}(892) \rightarrow K^- \pi^+] \pi^-$	6.45 ± 0.71	5.35 ± 0.59	5.73 ± 0.14	1.42
	7.20 ± 0.90	5.98 ± 0.75		
$\bar{B}^0 \rightarrow [\bar{K}^{*-}(892) \rightarrow \bar{K}^0 \pi^-] \pi^+$	5.60 ± 0.93	4.65 ± 0.77	5.42 ± 0.16	1.09
	$\frac{2}{3}(11.7 \pm 1.30)$	6.47 ± 0.72		

Table: Direct CP asymmetries

Decay mode	exp. (%)	model (%)	model (%) [$c_i^D \equiv 0$]
$B^- \rightarrow [\bar{K}^{*0}(892) \rightarrow K^- \pi^+] \pi^-$	-14.9 ± 6.8	-2.5 ± 1.3	1.4
	3.2 ± 5.4		
$B^- \rightarrow [\bar{K}_0^*(1430) \rightarrow K^- \pi^+] \pi^-$	7.6 ± 4.6		
$B^- \rightarrow (K^- \pi^+)_S \pi^-$	3.2 ± 4.6	5.4 ± 1.0	0.2
$\bar{B}^0 \rightarrow [\bar{K}^{*0}(892) \rightarrow \bar{K}^0 \pi^-] \pi^+$	-14 ± 12	-19.6 ± 3.0	6.1
$\bar{B}^0 \rightarrow (\bar{K}^0 \pi^-)_S \pi^+$	17 ± 26	-0.2 ± 1.3	-1.7

Scientific achievements:

J. P. Dedonder, A. Furman, R. Kaminski, L. Lesniak and B. Loiseau,
“*S-, P- and D-wave final state interactions and CP violation in $B^\pm \rightarrow \pi^\pm \pi^\mp \pi^\pm$ decays*”,
Acta Phys. Polon. B **42** (2011), 2013
49 citations, 31 pages

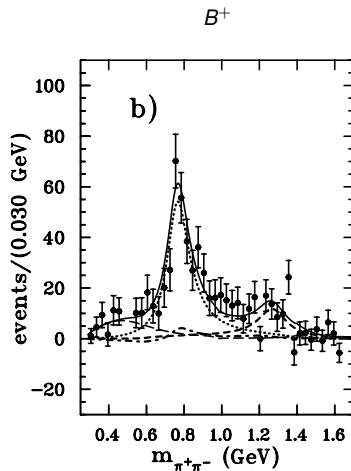
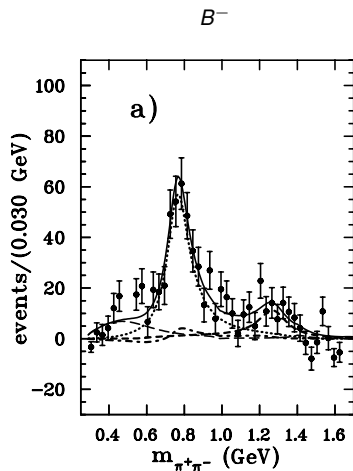
Method:

- QCDF approach,
- Unitary FSI,
- The pion scalar form factor - from a unitary relativistic coupled-channel model including $\pi\pi$, $K\bar{K}$ and $(2\pi)(2\pi)$ interactions,
- The pion vector form factor from a Belle analysis of $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ data,
- Three partial waves: *S*, *P* and *D*,
- The $B^\pm \rightarrow \pi^\pm \pi^\mp \pi^\pm$ BABAR data are fitted using only five parameters

Results (New):

- Sizable contribution of the *S* wave just above the $\pi\pi$ threshold and a significant interference between the *S* and *P* waves under the $\rho(770)$ peak (not seen in exp.),
- *B* to $f_2(1270)$ transition form factor $F^{Bf_2}(m_\pi^2)$ and a unified unitary description of the contribution of $f_0(600)$, $f_0(980)$ and $f_0(1400)$

Examples of our fits to BABAR data:



Scientific achievements:

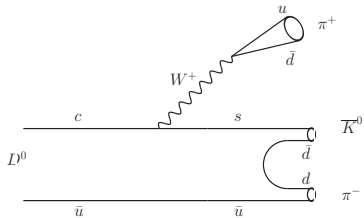
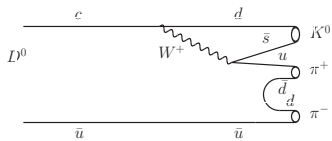
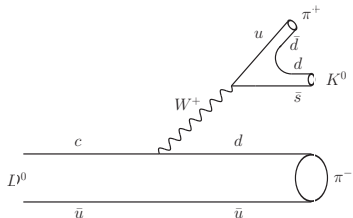
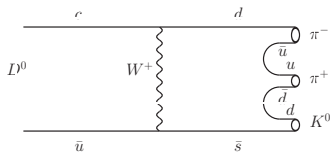
J. P. Dedonder, R. Kaminski, L. Lesniak and B. Loiseau,
“Dalitz plot studies of $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays in a factorization approach”,
Phys. Rev. D **89** (2014) no.9, 094018
31 citations, 34 pages

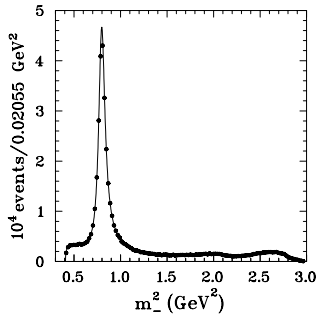
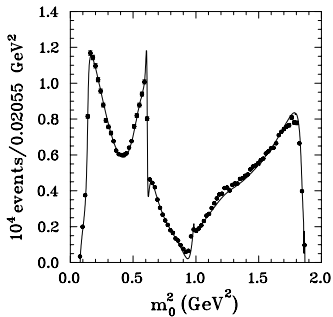
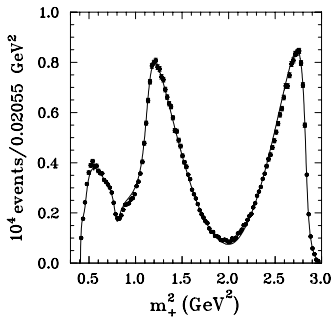
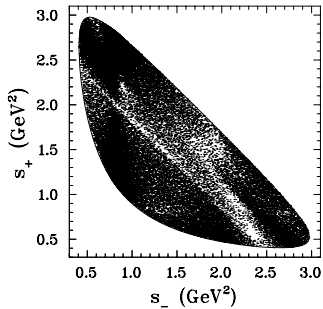
Method:

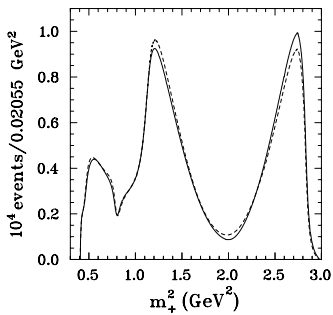
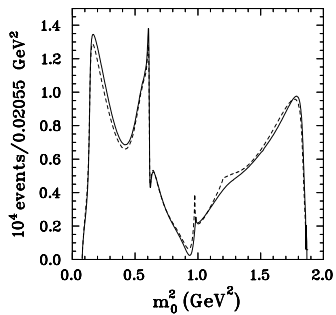
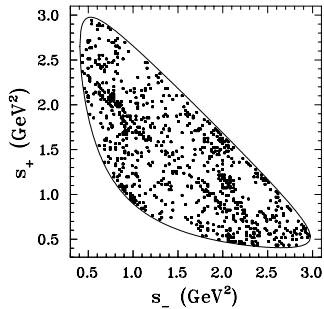
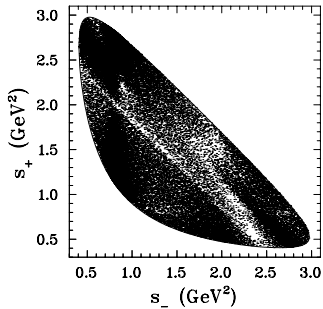
- Tree and annihilation amplitudes fitted to high-statistics Belle and BABAR data,
- The $K\pi$ and $\pi\pi$ scalar and vector form factors and relativistic Breit-Wigner formulae for the D waves,
- A combined χ^2 fit to: a Belle Dalitz plot, the total \mathcal{B}^{exp} and to the $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$ decay data with 33 free parameters,
- Another set of parameters fits the BABAR Collaboration Dalitz plot model,

Results:

- The parameters of both fits are close, the branching fractions of the dominant channels compare well with those of the isobar Belle or BABAR models,
- The unitary $K\pi$ and $\pi\pi$ scalar form factors satisfying analyticity and chiral symmetry constraints are constrained by the Dalitz plot analysis,
- Our $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decay amplitude could be a useful input for determinations of D^0 - \bar{D}^0 mixing parameters and of the CKM angle γ (or ϕ_3).







Scientific achievements:

D. Boito, J. P. Dedonder, B. El-Bennich, R. Escribano, R. Kaminski, L. Lesniak and B. Loiseau,

“Parametrizations of three-body hadronic B- and D-decay amplitudes in terms of analytic and unitary meson-meson form factors”,

Phys. Rev. D **96** (2017) no.11, 113003

30 citations, 38 pages

- We introduce parametrizations of hadronic three-body B and D weak decay amplitudes that can be readily implemented in experimental analyses and are a sound alternative to the simplistic and widely used sum of Breit-Wigner type amplitudes, also known as the isobar model.
- unitary S - and P -wave $\pi\pi$, πK and $K\bar{K}$ form factors fulfilling fundamental properties of QFT amplitudes such as analyticity and unitarity (in agreement with the low-energy behaviour of QCD) chiral symmetry and asymptotic QCD derived from sets of coupled-channel equations using T -matrix elements constrained by phase shifts and inelasticities.
- Explicit amplitude expressions for the decays $B^\pm \rightarrow \pi^+\pi^-\pi^\pm$, $B \rightarrow K \pi^+\pi^-$, $B^\pm \rightarrow K^+K^-K^\pm$, $D^+ \rightarrow \pi^-\pi^+\pi^+$, $D^+ \rightarrow K^-\pi^+\pi^+$, $D^0 \rightarrow K_S^0 \pi^+\pi^-$ and $D^0 \rightarrow K_S^0 K^+K^-$

Scientific achievements:

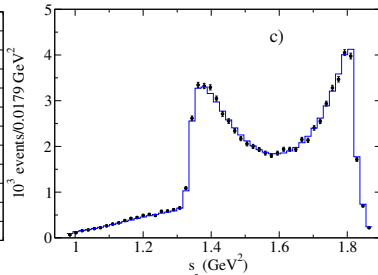
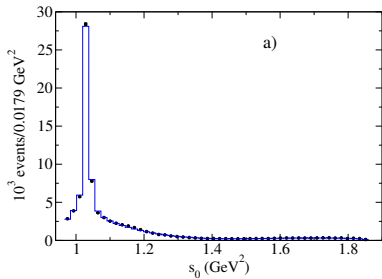
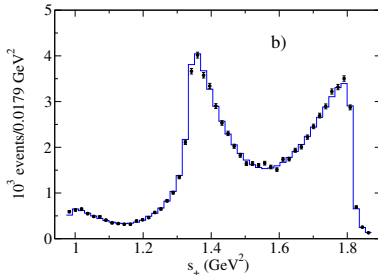
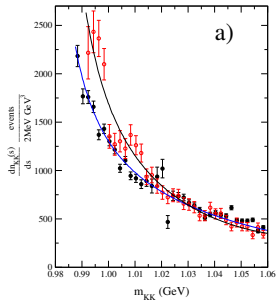
J. P. Dedonder, R. Kamiński, L. Leśniak and B. Loiseau,
“Dalitz plot studies of $D^0 \rightarrow K_S^0 K^+ K^-$ decays in a factorization approach”,
Phys. Rev. D **103** (2021) no.11, 114028,
3 citations, 31 pages

Method:

- BABAR data and QCDF,
- tree amplitudes: ($D^0 \rightarrow 2K$), annihilation ones (W -exchange),
- 19 free parameters

Results (New):

- i) the dominance of annihilation amplitudes,
 - ii) a large dominance of the $f_0(980)$ meson in the near threshold $K^+ K^-$, and
 - iii) a sizable B to the $[\rho(770)^+ + \rho(1450)^+ + \rho(1700)^+] K_S^0$ FSI.
- two alternative fits using the scalar-kaon form factors calculated from the Muskhelishvili-Omnès dispersion relation approach are shown.
- $K^+ K^-$ and $\bar{K}^0 K^+$ S -wave effective mass squared distributions are significantly different (model-independent manner) \rightarrow $f_0(980)$ resonance must be included.

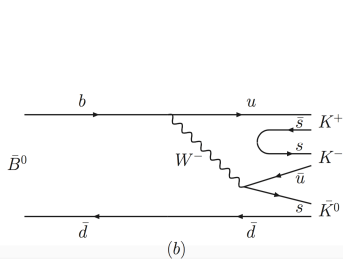
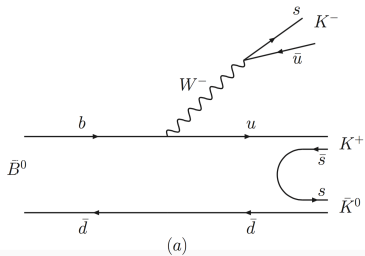


Article to be published asap (more than 95% done):

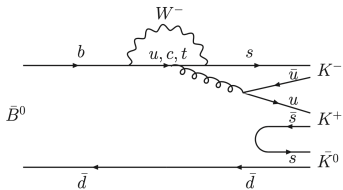
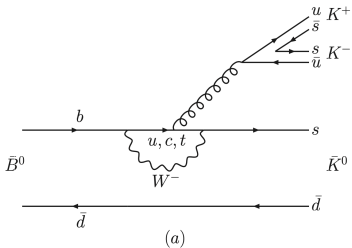
J. P. Dedonder, R. Kamiński, L. Leśniak and B. Loiseau, B. Moussallam, P. Żenczykowski,
“Amplitude analysis of $\bar{B}^0 \rightarrow K_S^0 K^+ K^-$ decays in a quasi-two-body QCD factorization approach”, 35.... pages (kind of review)

- The *BABAR* and Belle data are analyzed in QCDF,
- Decay amplitude: from unitary final-state meson-meson form factors,
- parametrization can (should?) replace the sum of Breit-Wigner type amplitudes (isobar model),
- model reproduce, in a quite reasonable way, the available experimental \mathcal{B}_{total} ($\chi^2 < 1$) together with \mathcal{B}_i
- With 12 free parameters our model fits 422 data with a $\chi^2/ndf \approx 1.3$
- $\sum \mathcal{B}_i = 105\%$ (instead of $\approx 130\%$ in exp. analyses),
- Calculations are done in 100%, text in $\approx 95\%$

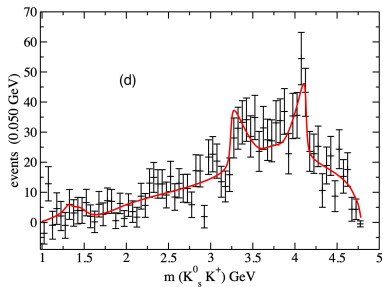
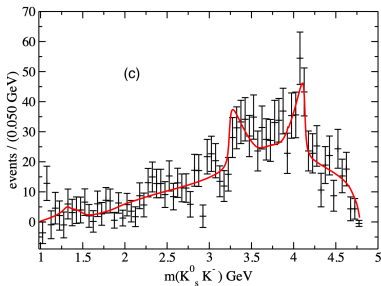
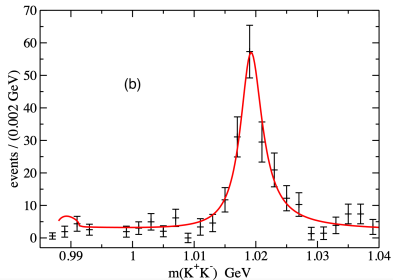
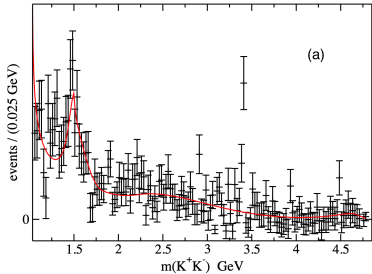
Tree diagrams:



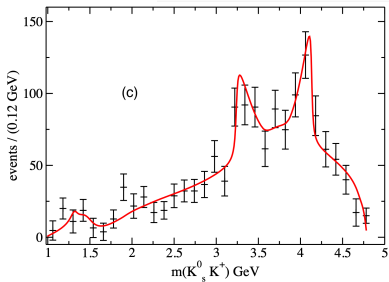
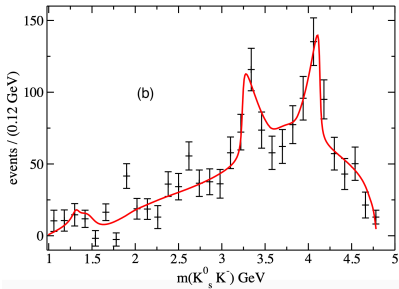
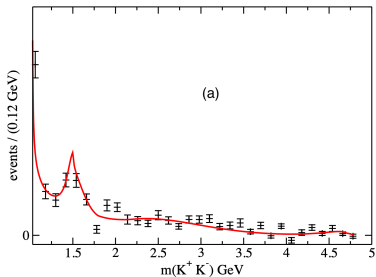
Penguin diagrams:



Fit to the Belle data:



Fit to the BABAR data:



Scientific plans for 2024:

Phenomenological analyses of high precision B and D decay data and tests of the Standard Model

Work plan:

- Work on $\bar{B}^0 \rightarrow K_S^0 K^+ K^-$ decays will be sent for publication before 01.01.2024 or in January 2024,
- Using our approach to analyze the available data on the $B^\pm \rightarrow K^+ K^- \pi^\pm$ decay,
- Checking whether in our model there is some CP asymmetry in the Dalitz plot,
- Phenomenological analyses of high precision rare and semi-leptonic B and D decay data measured by LHCb and Belle II collaborations
- Unbinned angular analysis of $B \rightarrow D^*(D\pi)l\nu_l$ to measure the Wilson coefficient C_{V_R} of the right-handed vector current. CKM parameter V_{cb} ... B anomaly ...

Analyses of high precision rare and semi-leptonic B and D decay data

"Challenges in Semileptonic B Decays" by P. Gambino et al., Eur.Phys.J.C 80 (2020) 10, 966 (after MITP workshop held on April 9-13, 2018, in Mainz)

"Two of the elements of the Cabibbo-Kobayashi-Maskawa quark mixing matrix, $|V_{ub}|$ and $|V_{cb}|$, are extracted from semileptonic B decays. The results of the B factories, analysed in the light of the most recent theoretical calculations, remain puzzling, because for both $|V_{ub}|$ and $|V_{cb}|$ the exclusive and inclusive determinations are in clear tension. Further, measurements in the τ channels at Belle, Babar, and LHCb show discrepancies with the Standard Model predictions, pointing to a possible violation of lepton flavor universality. LHCb and Belle II have the potential to resolve these issues in the next few years"

- Our method: either application of **QCD factorization** (LLK group) or **weak effective theory** with Hamiltonian expressed in terms of left-handed and right-handed vector currents (Emi Kou).
- Expected issues:
 - constraints to the Standard Model parameters,
 - a more accurate extraction of the CKM phase γ - Dalitz-plot amplitude analysis,
 - better determination of the strong interaction meson-meson amplitudes
- expected to be done at future B physics experiments - Belle II with large statistics

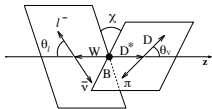
New physics search in $B \rightarrow D^* l \bar{\nu}_l$ angular observable

T. Kappor, Z.R. Huang, E. arXiv:2106.13855

- Differential decay rate of $B \rightarrow D^* l \bar{\nu}_l$ with Right-Handed current (C_{VR})

$$\frac{d\Gamma(\bar{B} \rightarrow D^*(\rightarrow D\pi) \ell^- \bar{\nu}_\ell)}{dw d\cos\theta_V d\cos\theta_\ell d\chi} = \frac{6m_B m_{D^*}^2}{8(4\pi)^4} \sqrt{w^2 - 1} (1 - 2wr + r^2) G_F^2 |V_{cb}|^2 \times \mathcal{B}(D^* \rightarrow D\pi)$$

$$\times \left\{ \begin{aligned} & J_{1s} \sin^2 \theta_V + J_{1c} \cos^2 \theta_V + (J_{2s} \sin^2 \theta_V + J_{2c} \cos^2 \theta_V) \cos 2\theta_\ell \\ & + J_3 \sin^2 \theta_V \sin^2 \theta_\ell \cos 2\chi \\ & + J_4 \sin 2\theta_V \sin 2\theta_\ell \cos \chi + J_5 \sin 2\theta_V \sin \theta_\ell \cos \chi \\ & + (J_{6s} \sin^2 \theta_V + J_{6c} \cos^2 \theta_V) \cos \theta_\ell \\ & + J_7 \sin 2\theta_V \sin \theta_\ell \sin \chi + J_8 \sin 2\theta_V \sin 2\theta_\ell \sin \chi \\ & + J_9 \sin^2 \theta_V \sin^2 \theta_\ell \sin 2\chi \end{aligned} \right\}$$



$C_{VR} \neq 0 \Rightarrow$ New physics!

$J_{1s} = \frac{3}{2}(H_+^2 + H_-^2)(C_{VL} ^2 + C_{VR} ^2) - 6H_+H_- \text{Re}[C_{VL} C_{VR}^*]$	$J_5 = -2(H_+H_0 - H_-H_0)(C_{VL} ^2 - C_{VR} ^2)$
$J_{1c} = 2H_0^2(C_{VL} ^2 + C_{VR} ^2) - 2\text{Re}[C_{VL} C_{VR}^*]$	$J_{6s} = -2(H_+^2 - H_-^2)(C_{VL} ^2 - C_{VR} ^2)$
$J_{2s} = \frac{1}{2}(H_+^2 + H_-^2)(C_{VL} ^2 + C_{VR} ^2) - 2H_+H_- \text{Re}[C_{VL} C_{VR}^*]$	$J_{6c} = 0$
$J_{2c} = -2H_0^2(C_{VL} ^2 + C_{VR} ^2) - 2\text{Re}[C_{VL} C_{VR}^*]$	$J_7 = 0$
$J_3 = -2H_+H_- (C_{VL} ^2 + C_{VR} ^2) + 2(H_+^2 + H_-^2) \text{Re}[C_{VL} C_{VR}^*]$	$J_8 = 2(H_+H_0 - H_-H_0) \text{Im}[C_{VL} C_{VR}^*]$
$J_4 = (H_+H_0 + H_-H_0)(C_{VL} ^2 + C_{VR} ^2) - 2\text{Re}[C_{VL} C_{VR}^*]$	$J_9 = 2(H_+^2 - H_-^2) \text{Im}[C_{VL} C_{VR}^*]$

The B->D* form factor from latest Lattice computations

Fermilab-MILK arXiv:2105.14019
 JLQCD arXiv:2306.05657
 HPQCD arXiv:2304.03137

- The helicity amplitudes $H_{\pm,0}$ contain B->D* form factors (FF)
- Lattice QCD groups determined, all FFs necessary to describe the B->D* l nu decay, including their w dependence.
- In lattice QCD method, the w dependence of FFs is obtained by chiral extrapolation. For example, the Fermilab group provided the FF at different point of w and correlation matrix.

$$\chi_{\text{latt}}^2(\vec{v}_0) = (\vec{v}_0 - \vec{v}_0^{\text{latt}})V^{-1}(\vec{v}_0 - \vec{v}_0^{\text{latt}})^T$$

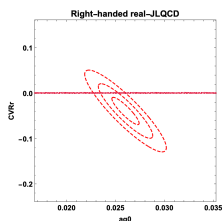
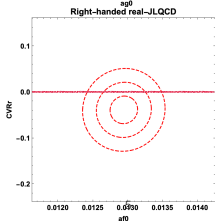
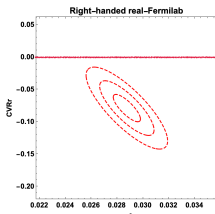
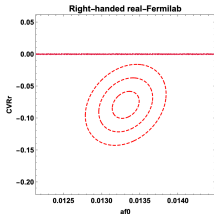
TABLE XX: Synthetic data of the chiral-continuum extrapolation for the form factors used in the z expansion with their correlation matrix.

Value	Correlation Matrix												
	$f(1.03)$	$g(1.03)$	$\mathcal{F}_1(1.03)$	$\mathcal{F}_2(1.03)$	$f(1.10)$	$g(1.10)$	$\mathcal{F}_1(1.10)$	$\mathcal{F}_2(1.10)$	$f(1.17)$	$g(1.17)$	$\mathcal{F}_1(1.17)$	$\mathcal{F}_2(1.17)$	
$f(1.03)$	5.77(11)	1.0000	0.1156	0.9885	0.6710	0.9247	0.1029	0.8596	0.6518	0.3743	0.0723	0.5229	0.4407
$g(1.03)$	0.371(14)	0.1156	1.0000	0.1304	0.3489	0.1245	0.9354	0.1564	0.3253	0.1057	0.6819	0.1373	0.2375
$\mathcal{F}_1(1.03)$	18.73(38)	0.9885	0.1304	1.0000	0.7304	0.9084	0.1153	0.9065	0.7204	0.5616	0.0816	0.5753	0.4991
$\mathcal{F}_2(1.03)$	2.175(70)	0.6710	0.3489	0.7304	1.0000	0.6024	0.3259	0.7364	0.9346	0.3584	0.2449	0.4923	0.6078
$f(1.10)$	5.49(12)	0.9247	0.1245	0.9084	0.6024	1.0000	0.1781	0.9031	0.6784	0.7973	0.2801	0.6881	0.5825
$g(1.10)$	0.330(14)	0.1029	0.9354	0.1153	0.3259	0.1781	1.0000	0.1954	0.3664	0.2416	0.8429	0.2442	0.3480
$\mathcal{F}_1(1.10)$	17.52(45)	0.8596	0.1564	0.9065	0.7364	0.9031	0.1954	1.0000	0.8430	0.6986	0.2002	0.8116	0.7270
$\mathcal{F}_2(1.10)$	1.912(69)	0.6518	0.3253	0.7204	0.9346	0.6784	0.3664	0.8430	1.0000	0.5238	0.3358	0.6981	0.7960
$f(1.17)$	5.23(17)	0.5743	0.1057	0.5616	0.3584	0.7973	0.2416	0.6986	0.5238	1.0000	0.3769	0.8364	0.7089
$g(1.17)$	0.290(17)	0.0723	0.6819	0.0816	0.2449	0.2001	0.8429	0.2002	0.3358	0.3769	1.0000	0.3571	0.4456
$\mathcal{F}_1(1.17)$	16.46(70)	0.5229	0.1373	0.5753	0.4923	0.6881	0.2442	0.8116	0.6981	0.8364	0.3571	1.0000	0.9218
$\mathcal{F}_2(1.17)$	1.692(91)	0.4407	0.2375	0.4991	0.6078	0.5825	0.3480	0.7270	0.7960	0.7089	0.4456	0.9218	1.0000

Right-Handed fit including lattice QCD data

We fit the Belle experimental data and lattice QCD data simultaneously

T. Kappor, Z.R. Huang, E.K.
arXiv:2311.xxxxx
(to be submitted soon)



We found some hint of non-zero C_{VR}
though it is controversial:
Fermilab ~ 4 sigma
JLQCD ~ 1.5 sigma
A more detail investigation needed!

Thanks!