Exotic Nuclear Structure Mechanisms and Symmetries and their Identifications through Theory and Experiment

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COPIN Project 23-157

II. Identification of the collaboration	
Title of the collaboration	Exotic nuclear structure mechanisms and symmetries â€" their identifications through theory and experiment
Number of the collaboration	23-157
IN2P3 spokesperson	Jerzy DUDEK
COPIN spokesperson	Irene DEDES
Scientific Domain	Nuclear Physics

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What does *Exotic Nuclear Symmetries* stand for?

• Symmetries from the *world of molecules* but detected in subatomic physics, such as Tetrahedral (T_d) and Octahedral (O_h) Symmetries → **High Rank Symmetries**

Why Are We Interested in *Molecular Symmetries* in Subatomic Physics ?

• Nuclear symmetries generate unprecedented **degeneracies** in both **individual-nucleonic** and **collective-rotational levels: New Issues !**

• Thus – Implied totally new spectroscopy rules in subatomic physics

• We found the first experimental confirmation of T_d in ^{152}Sm nucleus

• Presence of an unprecedented class of exotic shape-isomeric states

FURTHER CONSEQUENCES for SUBATOMIC PHYSICS

• New highway towards exotic nuclei: **Isomers** living longer than the ground-state
• Astrophysics: **New magic numbers** for the nucleosynthesis

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Main Scope of Our Research

• Give the theoretical explanations to the experimental nuclear structure phenomena observed, as well as predicting the still unknown

How do we perform our studies?

• We describe the nuclear interior, i.e. *nuclear structure*, with a simple but very reliable and powerful theory called: **The Nuclear Mean-Field Theory**

• We combine contemporary **mathematical tools** of group theory, inverse problem theory and graph-theory with phenomenological nuclear mean-field theory

• One of the most important strategies: Making sure the theory we use is reliable, offering realistic, experiment comparable results for many nuclei.

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Part 1

Remarks about Our Choice of Theory Approach:

Phenomenological Mean Field

Deformed Universal Woods-Saxon Hamiltonian

Reminder about standard defintions

Introducing Woods-Saxon Hamiltonian

- We use the phenomenological **Woods-Saxon Hamiltonian** with the so-called **'universal'** parameterisation

$\Rightarrow$ fixed set of parameters for thousands of nuclei!

- **Central Potential**

$$\mathcal{V}^{\text{WS}}_{\text{cent}} = \frac{V_c}{1 + \exp\left[\text{dist}_\Sigma(\vec{r}; r_c)/a_c\right]}$$

- **Spin-Orbit Potential**

$$\mathcal{V}^{\text{WS}}_{\text{SO}} = \frac{2\hbar\lambda_{so}}{(2mc)^2}\left[(\vec{\nabla}V^{\text{WS}}_{\text{SO}}) \wedge \hat{p}\right] \cdot \hat{s}, \ \text{with } V^{\text{WS}}_{\text{SO}} = \frac{V_o}{1 + \exp[\text{dist}_\Sigma(\vec{r}, r_{so})/a_{so}]}$$

- **Isospin distinction** ($+ \leftrightarrow$ protons) and ($- \leftrightarrow$ neutrons)

$$V_c = V_o\left[1 \pm \kappa_c \frac{N-Z}{N+Z}\right]; \quad \lambda_{so} = \lambda_o\left[1 \pm \kappa_{so} \frac{N-Z}{N+Z}\right]$$

- **This potential depends *only* on two sets of 6 parameters $\leftrightarrow$ Mass Table**

$$\boxed{\{V_c, r_c, a_c; \lambda_{so}, r_{so}, a_{so}\}_{\pi,\nu}} \Leftrightarrow \boxed{\{V_o, \kappa_c, r_c^{\pi,\nu}, a_c^{\pi,\nu}; \lambda_o, \kappa_{so}, r_{so}^{\pi,\nu}, a_{so}^{\pi,\nu}\}}$$

Part 2

Selected Molecular Symmetries in Atomic Nuclei

Example: So-called High-Rank*) Symmetries

Tetrahedral T_d and Octahedral O_h

*) The only ones with 4D irreducible spinor representations – 4-fold nucleonic degeneracies

Tetrahedral Symmetry: Spherical-Harmonic Basis

- Given a nuclear surface, Σ: $R(\vartheta,\varphi) = R_o\, c(\{\alpha\}) \left[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\vartheta,\varphi)\right]$
- Only special combinations of spherical harmonics may form a basis for surfaces with tetrahedral symmetry and only odd-order except 5

Three Lowest Order Solutions: **Rank ↔ Multipolarity λ**

$$\lambda = 3: \quad t_1 \equiv \alpha_{3,\pm 2}$$

$\lambda = 5:$ no solution possible

$$\lambda = 7: \quad t_2 \equiv \alpha_{7,\pm 2} \quad \text{and} \quad \alpha_{7,\pm 6} = -\sqrt{\tfrac{11}{13}} \cdot \alpha_{7,\pm 2}$$

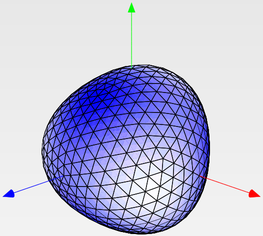
$$\lambda = 9: \quad t_3 \equiv \alpha_{9,\pm 2} \quad \text{and} \quad \alpha_{9,\pm 6} = +\sqrt{\tfrac{28}{198}} \cdot \alpha_{9,\pm 2}$$

- Problem presented in detail in:

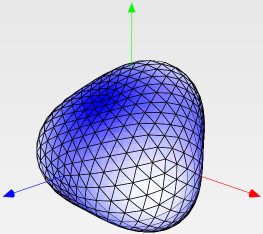
J. Dudek, J. Dobaczewski, N. Dubray, A. Góźdź, V. Pangon and N. Schunck, Int. J. Mod. Phys. E16, 516 (2007) [516-532].

Nuclear Tetrahedral Shapes – 3D Examples

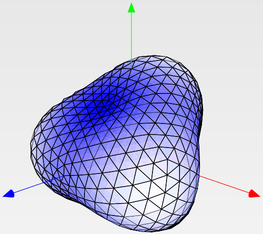
Illustrations below show the tetrahedral-symmetric surfaces at three increasing values of rank $\lambda = 3$ deformations $\alpha_{32} \equiv t_1$: 0.1, 0.2 and 0.3



$\alpha_{32} \equiv t_1 = 0.1$



$\alpha_{32} \equiv t_1 = 0.2$



$\alpha_{32} \equiv t_1 = 0.3$

Observations:

- There are infinitely many tetrahedral-symmetric surfaces
- Nuclear ‘pyramids’ do not resemble pyramids very much!

OBSERVATION:

Tetrahedral symmetry group, T_d,
is a sub-group of the octahedral one, O_h

Octahedral Symmetry: Spherical-Harmonic Basis

- Given a nuclear surface, Σ: $R(\vartheta,\varphi) = R_o\, c(\{\alpha\})\,\left[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\vartheta,\varphi)\right]$

- Only special combinations of spherical harmonics may form a basis for surfaces with octahedral symmetry and only in even-orders $\lambda \geq 4$

Three Lowest Order Solutions: **Rank ↔ Multipolarity λ**

$$\lambda = 4: \quad o_1 \equiv \quad \alpha_{40} \quad \text{and} \quad \alpha_{4,\pm4} = -\sqrt{\tfrac{5}{14}} \cdot \alpha_{40}$$

$$\lambda = 6: \quad o_2 \equiv \quad \alpha_{60} \quad \text{and} \quad \alpha_{6,\pm4} = -\sqrt{\tfrac{7}{2}} \cdot \alpha_{60}$$

$$\lambda = 8: \quad o_3 \equiv \quad \alpha_{80} \quad \text{and} \quad \alpha_{8,\pm4} = \sqrt{\tfrac{28}{198}} \cdot \alpha_{80}$$

$$\text{and} \quad \alpha_{8,\pm8} = \sqrt{\tfrac{65}{198}} \cdot \alpha_{80}$$

- Problem presented in detail in:

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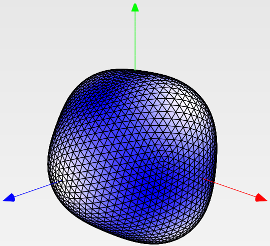
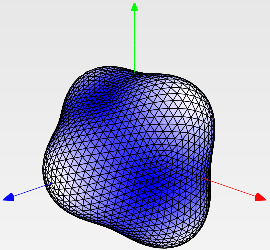
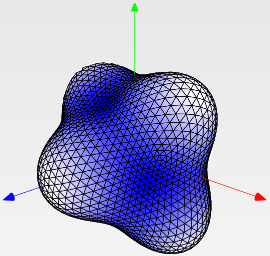


$o_1 = 0.1$



$o_1 = 0.2$



$o_1 = 0.3$

Observations:

- There are infinitely many octahedral-symmetric surfaces
- Nuclear 'diamonds' do not resemble diamonds very much!

Symmetries Are the Factors Determining Stability*) of Atomic Nuclei

*) *... by imposing hindrance mechanisms*

Symmetries Are the Factors Determining Stability*) of Atomic Nuclei

Nuclear mean field theory and group representation theory which are used in this research belong to the most powerful tools of nuclear structure theory arsenal

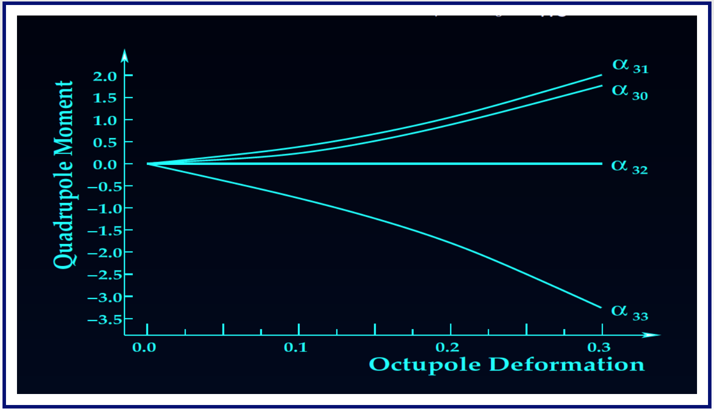
*) … *by imposing hindrance mechanisms*

Possible Measurable Signs of Nuclear Tetrahedral Symmetry

Quadrupole Moments vs. Pure Octupole Shapes

Indeed, for microscopically calculated quadrupole moments (W.S.)

$$Q_{20}(\alpha_{3\mu}) = \int \Psi^*_{WS}(\tau)\hat{Q}_{20}\Psi_{WS}(\tau)d\tau$$



Observe that $Q_{20}(\alpha_{32})$ vanishes identically at T_d-symmetric shapes

The Notion of Isomeric Bands

Similarly one demonstrates that tetrahedral shapes induce B(E1)=0

One shows that the analogous rules apply for octahedral symmetry

Once those symmetries are present one may expect the presence of numerous isomers since B(E2) and B(E1) at the exact tetrahedral and/or octahedral symmetry limits – vanish!

As the result, one expects series of long living (isomeric) states with unprecedented parabolic energy-spin relation

Isomers at: $E_I \propto I(I+1) \leftarrow$ **Isomeric Bands**

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Rotating High-Rank Symmetric Nuclei
Seen Through Group-Representation Theory

[Symmetry Properties of Quantum Rotors]

Tetrahedral Bands Are Not Like the Others!

As we have shown using the methods of the point-group representation theory that, for instance, rotational bands based on 0^+ “T_d ground-state” have the structure:

$$A_1: \quad 0^+, 3^-, 4^+, 6^+, 6^-, 7^-, 8^+, 9^+, 9^-, 10^+, 10^-, 11^-, 2\times 12^+, 12^-, \cdots$$

and NOT

$$\boldsymbol{I^\pi: \quad 0^+, 2^+, 4^+, 6^+, 8^+, 10^+, 12^+, \cdots}$$

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$$\boldsymbol{I^\pi : \quad 0^+, 2^+, 4^+, 6^+, 8^+, 10^+, 12^+, \cdots}$$

Similarly there are **no analogies** of the "octupole bands"

$$\boldsymbol{I^\pi : \quad 3^-, 5^-, 7^-, 9^-, 11^-, 13^-, 15^-, \cdots}$$

Quantum Rotors: Tetrahedral vs. Octahedral

- The tetrahedral T_d symmetry group has 5 irreducible representations
- The ground-state $I^{\pi} = 0^{+}$ belongs to A_1 representation given by:

$$\underbrace{\mathrm{A}_1 : \quad 0^+,\ 3^-,\ 4^+,\ \underbrace{(6^+, 6^-)}_{\text{doublet}},\ 7^-,\ 8^+,\ \underbrace{(9^+, 9^-)}_{\text{doublet}},\ \underbrace{(10^+, 10^-)}_{\text{doublet}},\ 11^-,\ \underbrace{2\times 12^+,\ 12^-}_{\text{triplet}},\ \cdots}_{\textbf{Forming a common parabola}}$$

- There are no states with spins $I = 1, 2$ and 5. We have parity doublets: $I = 6, 9, 10\ \ldots$, at energies: $E_{6^-} \approx E_{6^+}$, $E_{9^-} \approx E_{9^+}$, etc.

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- One shows the analogue structures for the octahedral $\mathrm{O_h}$ symmetry

$$\underbrace{A_{1g}:\ 0^+, 4^+, 6^+, 8^+, 9^+, 10^+, \ \ldots,\ \ I^\pi = I^+}_{\textbf{Forming a common parabola}}$$

$$\underbrace{A_{2u}:\ 3^-, 6^-, 7^-, 9^-, 10^-, 11^-, \ldots,\ \ I^\pi = I^-}$$

Experimental Data Selection for T_d

About criteria for the experimental data search

- Central condition followed: Nuclear states with exact high-rank symmetries produce neither dipole-, nor quadrupole moments

- Such states neither emit any collective/strong E1/E2 transitions nor can be fed by such transitions $\longrightarrow$ focus on the nuclear processes

- Therefore we decided to focus first of all on the nuclei which can be populated with a big number of nuclear reactions since we may expect that - in such nuclei - the states sought exist in the literature

- We had verified that the nucleus ^{152}Sm can be produced by about 25 nuclear reactions, whereas surrounding nuclei can be produced typically with about a dozen but usually much fewer reactions only

- Energy-wise – tetrahedral bands form regular sequences

$$E_I \propto AI^2 + BI + C$$

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Announcement of the Discovery – Part I

PHYSICAL REVIEW C VOLUME 97, 021302(R) FEBRUARY 2018

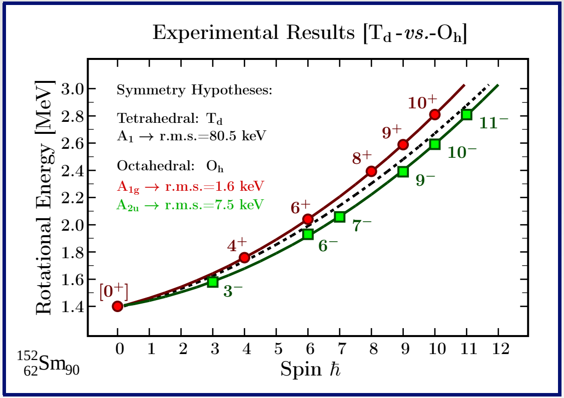
Spectroscopic criteria for identification of nuclear tetrahedral and octahedral symmetries: Illustration on a rare earth nucleus

J. Dudek, D. Curien, I. Dedes, K. Mazurek, S. Tagami, Y. R. Shimizu and T. Bhattacharjee

(Received 8 June 2017)

We formulate criteria for identification of the nuclear tetrahedral and octahedral symmetries and illustrate for the first time their possible realization in a rare earth nucleus ^{152}Sm. We use realistic nuclear mean-field theory calculations with the phenomenological macroscopic-microscopic method, the Gogny-Hartree-Fock-Bogoliubov approach, and general point-group theory considerations to guide the experimental identification method as illustrated on published experimental data. Following group theory the examined symmetries imply the existence of exotic rotational bands on whose properties the spectroscopic identification criteria are based. These bands may contain simultaneously states of even and odd spins, of both parities and parity doublets at well-defined spins. In the exact-symmetry limit those bands involve no E2 transitions. We show that coexistence of tetrahedral and octahedral deformations is essential when calculating the corresponding energy minima and surrounding barriers, and that it has a characteristic impact on the rotational bands. The symmetries in question imply the existence of long-lived shape isomers and, possibly, new waiting point nuclei-impacting the nucleosynthesis processes in astrophysics – and an existence of 16-fold degenerate particle-hole excitations.

Perfect Parabolas Represent Experimental Results

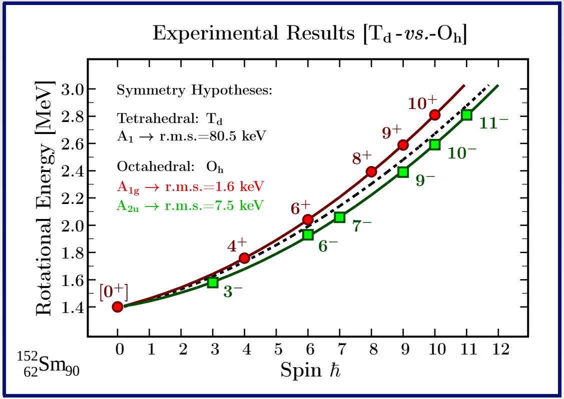


- Sequences represent **coexistence between tetrahedral and octahedral symmetries**

- Curves represent the parabolic fit and are *not* meant to guide the eye.

This is the first evidence of T_d (dashed) and O_h based on the experimental data

Perfect Parabolas Represent Experimental Results



FROM: **Spectroscopic criteria for identification of nuclear tetrahedral and octahedral symmetries: Illustration on a rare earth nucleus**

J. Dudek et al., PHYSICAL REVIEW C 97, 021302(R) (2018)

[DOI: https://doi.org/10.1103/PhysRevC.97.021302]

Part 3

About Exotic Shape-Instabilities in Actinides

Path to Exotic Symmetries: Begin with Spherical ^{208}Pb

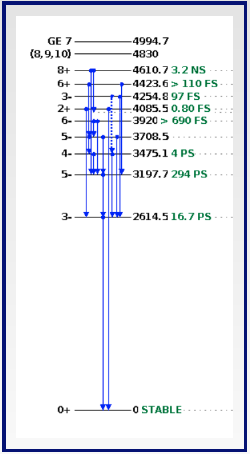
- Consider ^{208}Pb nucleus, doubly magic, among the most stable, spherical, ...

- The first excited state is an $I^\pi = 3^-$, traditionally associated with the pear-shape $Y_{\lambda=3,\mu=0}$-oscillations

- Other negative parity octupole modes are generated by multipolarities $Y_{\lambda=3,\mu\neq0}$

Multipolarity $\alpha_{\lambda=3,\mu}$	Point Group
α_{30}	$C_{\infty v}$
α_{31}	C_{2v}
α_{32}	T_d
α_{33}	D_{3h}

- One can demonstrate that these are the corresponding Point Groups

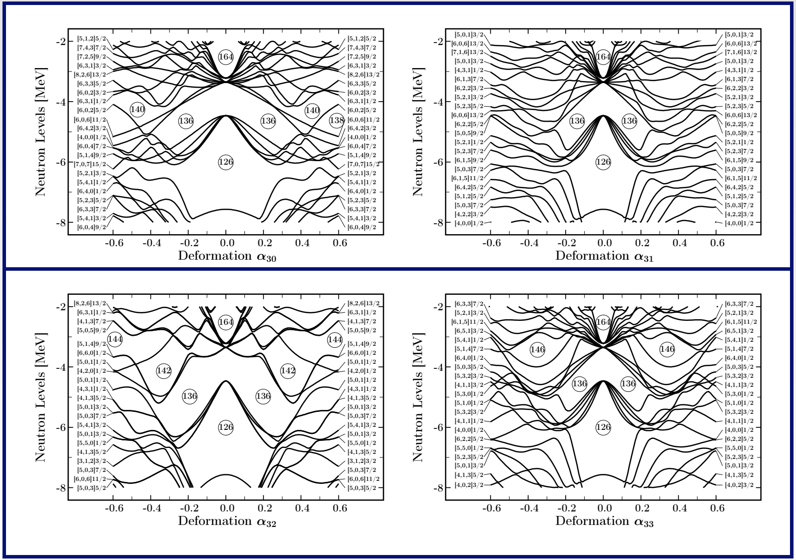


^{208}Pb Level Scheme from NNDC; 3^- state traditionally associated with the octupole (pear-shape) oscillations

What structural mechanisms are expected to bring the $I^{\pi} = 3^{-}$ vibrations
to the lowest position in the spectrum?

More generally,
what are the shell mechanisms responsible of
lowering the negative parity collective states?

Shell Structures at $N = 136 \rightarrow \alpha_{30}, \alpha_{31}, \alpha_{32}, \alpha_{33}$



We conclude that $N = 136$ plays the role of a special octupole magic-number and this – for all the 4 octupole multipolarities

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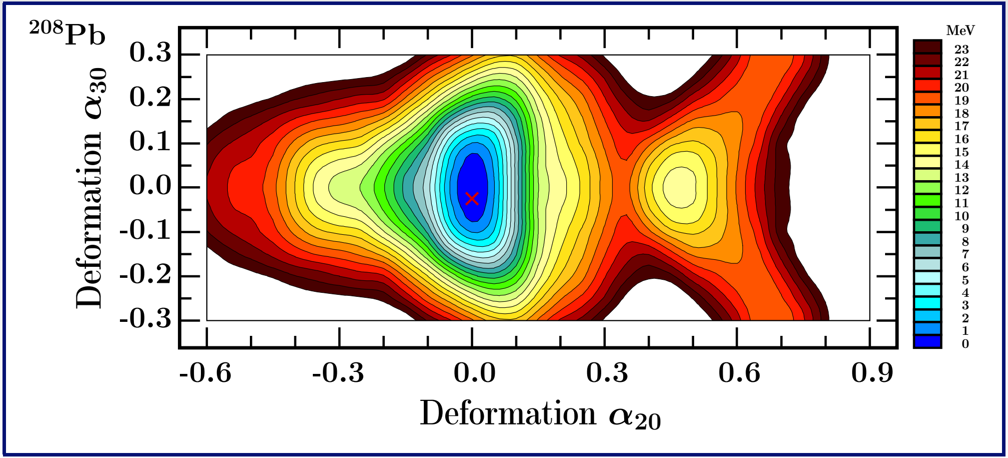
Consequences in terms of the nuclear structure[*)]

*) I. Hamamoto, B. Mottelson, H. Xie, and X. Z. Zhang, Z. Phys. D - Atoms, Molecules and Clusters 21, 163-175 (1991)

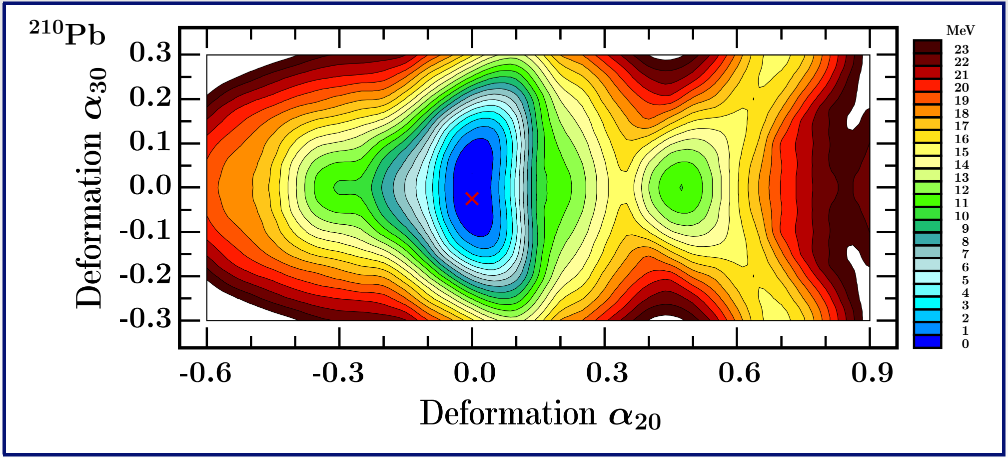
Evolution of Pear-Shape Instabilities: ^{208}Pb

- Total Nuclear Energy projected on the $(\alpha_{20}, \alpha_{30})$-plane minimised over $(\alpha_{22}, \alpha_{40})$ for ^{208}Pb



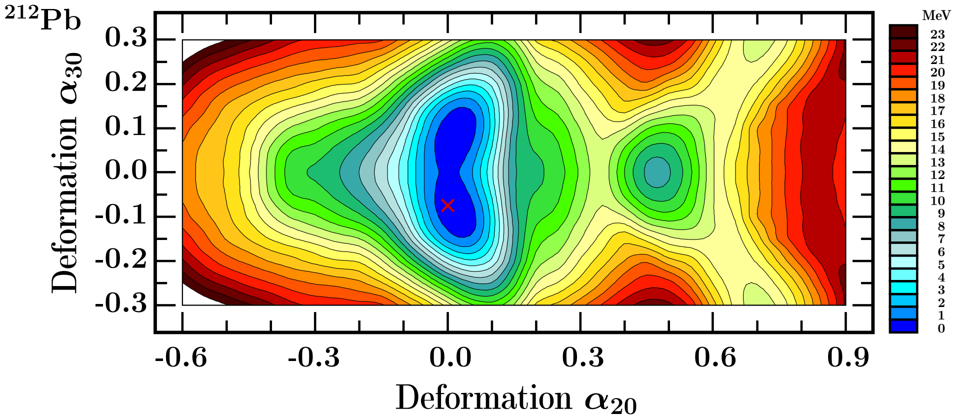
Evolution of Pear-Shape Instabilities: ^{210}Pb

- Projection on the $(\alpha_{20}, \alpha_{30})$-plane minimised over $(\alpha_{22}, \alpha_{40})$ for ^{210}Pb



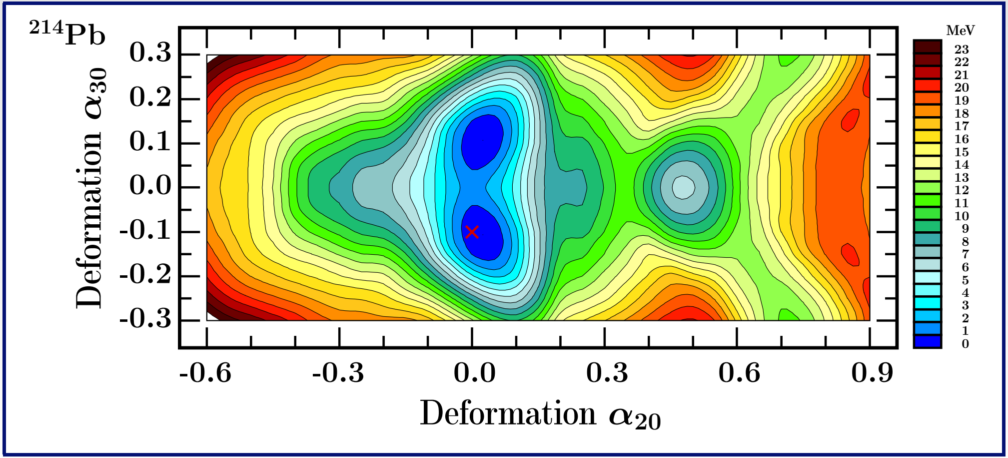
Evolution of Pear-Shape Instabilities: ^{212}Pb

- Projection on the $(\alpha_{20}, \alpha_{30})$-plane minimised over $(\alpha_{22}, \alpha_{40})$ for ^{212}Pb



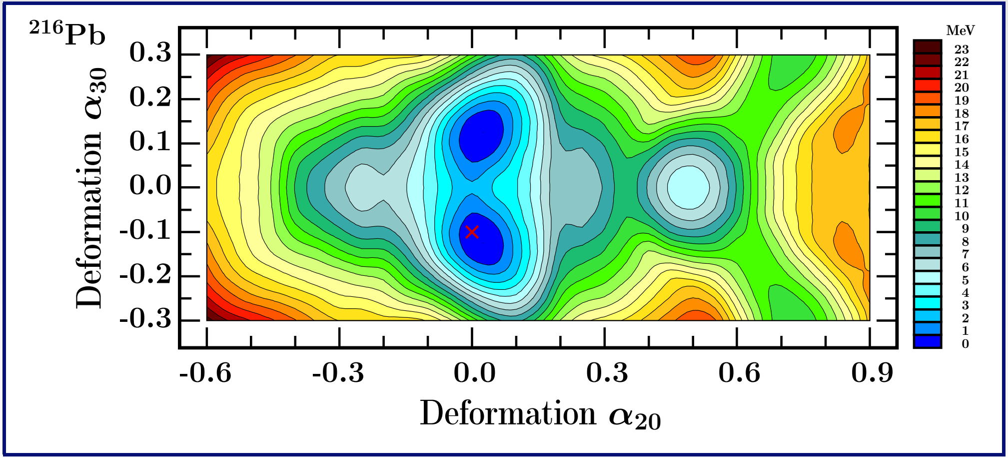
Evolution of Pear-Shape Instabilities: ^{214}Pb

- Projection on the $(\alpha_{20}, \alpha_{30})$-plane minimised over $(\alpha_{22}, \alpha_{40})$ for ^{214}Pb



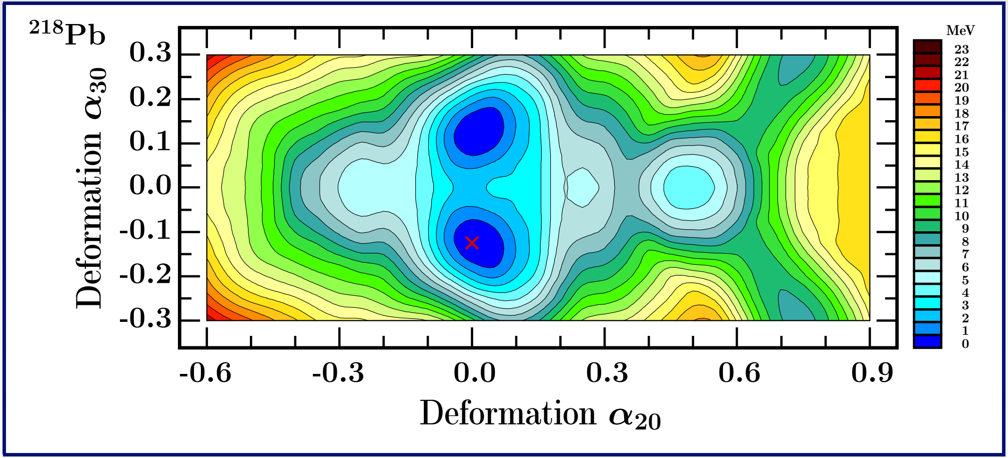
Evolution of Pear-Shape Instabilities: ^{216}Pb

- Projection on the $(\alpha_{20}, \alpha_{30})$-plane minimised over $(\alpha_{22}, \alpha_{40})$ for ^{216}Pb



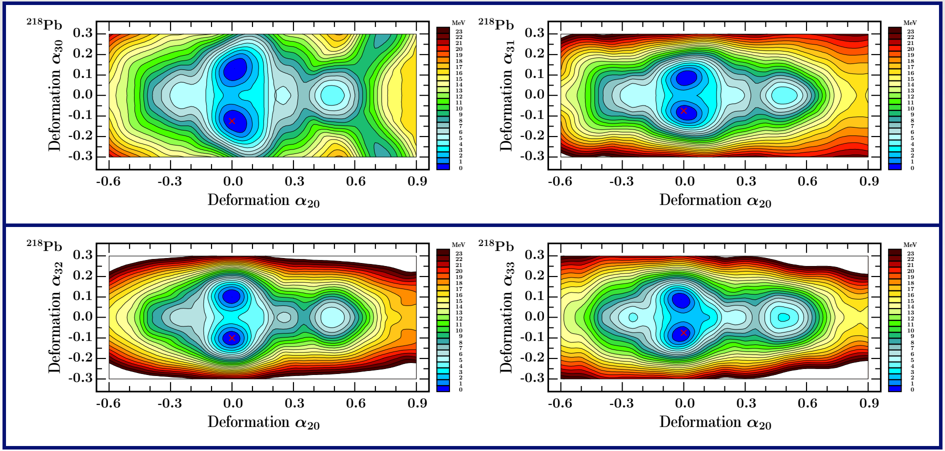
Evolution of Pear-Shape Instabilities: ^{218}Pb

- Projection on the $(\alpha_{20}, \alpha_{30})$-plane minimised over $(\alpha_{22}, \alpha_{40})$ for ^{218}Pb



Comparison: $\lambda = 3$ Susceptibility in ^{218}Pb Region

- Projection on the $(\alpha_{20}, \alpha_{3\mu})$-plane minimised over $(\alpha_{22}, \alpha_{40})$ for ^{218}Pb



Observations about Heavy Pb Isotopes

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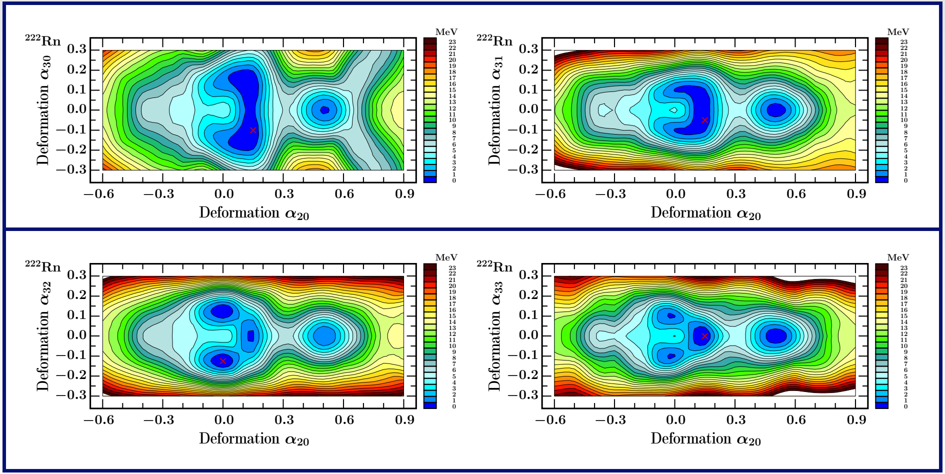
$\Longrightarrow$ We check the $Z > 82$ nuclei since they are easier to access experimentally

Exotic Symmetries for $Z > 82$ Nuclei: ^{222}Rn

- Projection on the $(\alpha_{20}, \alpha_{3\mu})$-plane minimised over $(\alpha_{22}, \alpha_{40})$



Observations

- Appearance of strongly pronounced octupole minima in nuclei with $Z > 82$, especially those close to $N = 136$
- In contrast to the Pb case, some of the octupole instabilities appear for $\alpha_{20} \neq 0.0$
- This favours the experimental identification of slightly broken tetrahedral symmetry since with $B(E2) \neq 0$ one can hope for profiting from the Germanium multi-detector systems and identify, even if weak, quadrupole transitions

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$\Rightarrow$ What are the induced exotic molecular symmetries? $\Leftarrow$

We use Point Group and Group-Representation Theories

Synthetic View of Octupole Instabilities

- The octupole-shape deformations include $\alpha_{\lambda=3,\mu=0,1,2,3}$ thus leading to 4 independent degrees of freedom (Note: minima obtained at $\alpha_{20}=0$)

$$\{\alpha_{30} \neq 0,\ \alpha_{31} \neq 0,\ \alpha_{32} \neq 0,\ \alpha_{33} \neq 0\}$$

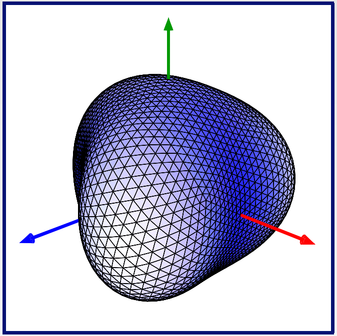
- One can demonstrate that they generate **Point-Group Symmetries:**

$C_{\infty v}$, C_{2v}, T_d, D_{3h}, respectively

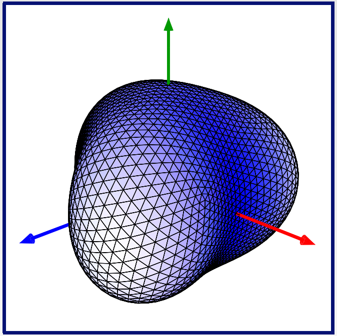
- It turns out that octupole static or dynamic state equilibria may lead to specific rotational band structures ⇒ what are these structures?

Molecular (Point-Group) Symmetries - $C_{2v} \Leftrightarrow \alpha_{31}$

- Symmetry induced by both $(\alpha_{31} \neq 0)$ and $(\alpha_{20} \neq 0, \alpha_{31} \neq 0)$



$\alpha_{31} = 0.25$



$\alpha_{20} = 0.15, \alpha_{31} = 0.25$

Nuclear C_{2v} Point Group Symmetry

Molecular (Point-Group) Symmetries - T_d & $D_{2d} \Leftrightarrow \alpha_{32}$

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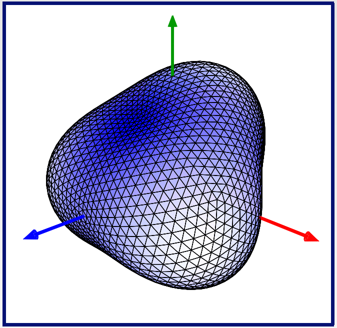
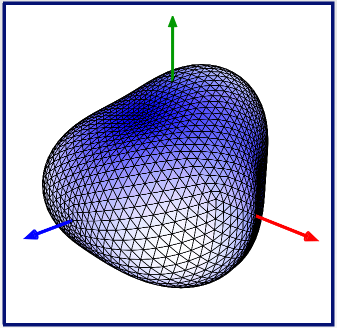


Tetrahedral T_d: $\alpha_{32} = 0.25$

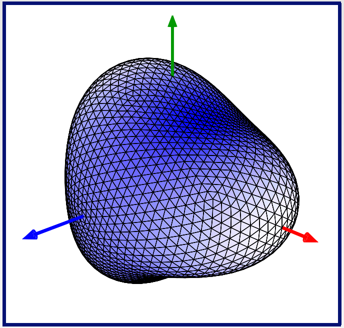


D_{2d}: $\alpha_{20} = 0.15, \alpha_{32} = 0.25$

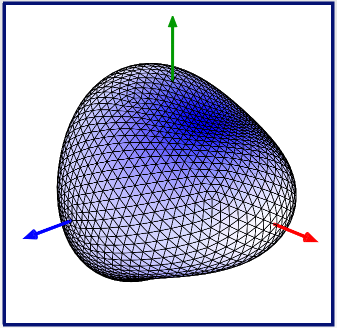
Nuclear T_d and D_{2d} Point Group Symmetries

Molecular (Point-Group) Symmetries - $D_{3h} \Leftrightarrow \alpha_{33}$

- Symmetry induced by both ($\alpha_{33} \neq 0$) and ($\alpha_{20} \neq 0, \alpha_{33} \neq 0$)



$\alpha_{33} = 0.25$



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Nuclear D_{3h} Point Group Symmetry

How to proceed once we know the point group representing a certain symmetry of interest?

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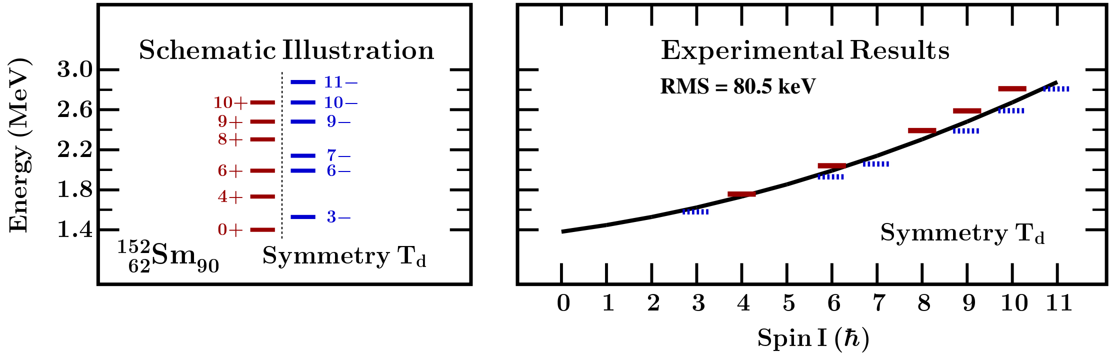
Suggestion: Examine rotational properties of concerned nuclei with the help of the group representation theory

Rotational Band Properties of Exotic Symmetries: T_d

The first tetrahedral symmetry evidence based on the experimental data



Tetrahedral Band : $I^{\pi}_{T_d}$ = $0^+, 3^-, 4^+, 6^\pm, 7^-, 8^+, 9^\pm, 10^\pm, 11^-, \ldots$

→ Published in: J. Dudek et al., PHYSICAL REVIEW C 97, 021302(R) (2018)

- The R.M.S. of the ground-state band is 15.18 keV

G.S.B. Predictions Overview: C_{2v}, D_{2d} and D_{3h}

- Group-theory prediction of the spin-parity structure of the $\mathbf{C_{2v}}$ g.s.b. spin-parity sequence for A_1-representation

$$C_{2v} \rightarrow A_1 : 0^+, 1^-, 2 \times 2^+, 2^-, 3^+, 2 \times 3^-, 3 \times 4^+, 2 \times 4^-, 2 \times 5^+, 3 \times 5^-, 4 \times 6^+, 4 \times 6^-, \cdots$$

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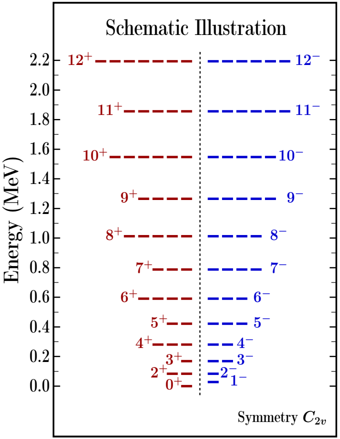
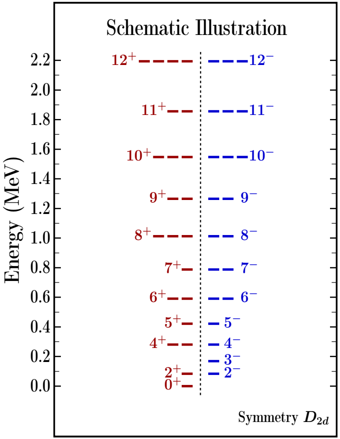
- **No $\Delta I = 2$ sequences !!**

Rotational Band Properties of Exotic Symmetries

- Each point group symmetry implies **specific degeneracy patterns**



C_{2v} : $(\alpha_{20}, \alpha_{31})$



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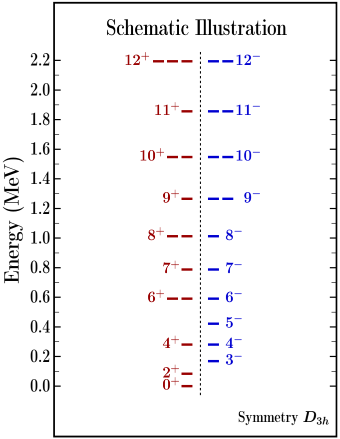

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- **Analysing NNDC experimental data for T_d symmetry in ^{152}Sm took 3 months of manual work**

- **Collecting experimental evidence via NNDC for C_{2v} in ^{236}U took 30 seconds of computer program*)**

*) I. Dedes in collaboration with M. Martin, Simon Fraser University, Canada

Experimental Data Selection for C_{2v}

About criteria for the experimental data search

$$C_{2v} \rightarrow A_1 : 0^+,\ 1^-, 2\times 2^+, 2^-, 3^+, 2\times 3^-, 3\times 4^+, 2\times 4^-, 2\times 5^+, 3\times 5^-, 4\times 6^+, 4\times 6^-, \cdots$$

- Avoid rotational bands generated by leading ellipsoidal geometry and characterised by strong $\Delta I = 2$ quadrupole transitions
- Identified yrast-trap or K-isomers and related axial symmetry non-collective particle-hole excitations should be eliminated
- Energy-wise – C_{2v} bands form regular sequences

$$E_I \propto AI^2 + BI + C$$

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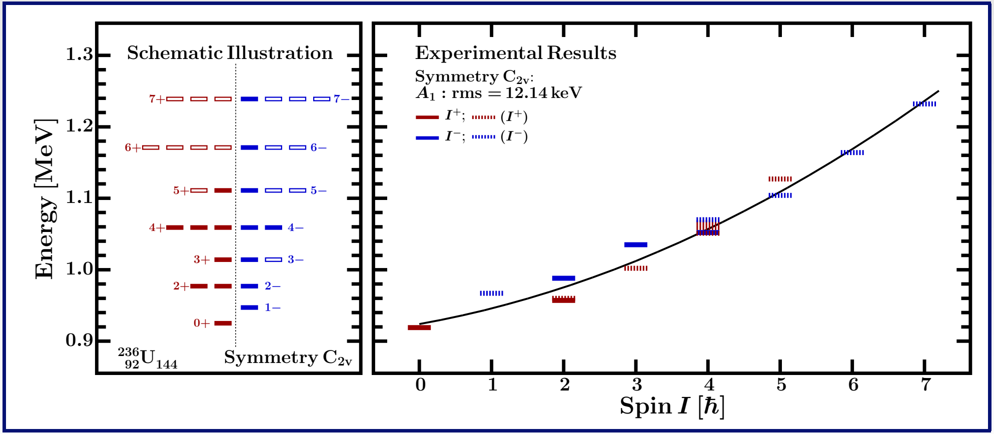
Experimental Identification - Recent Results : ^{236}U

- **Rotational band structure of a nucleus in a C_{2v}-symmetric configuration**

Attention: Experimental degeneracies for ^{236}U according to NNDC



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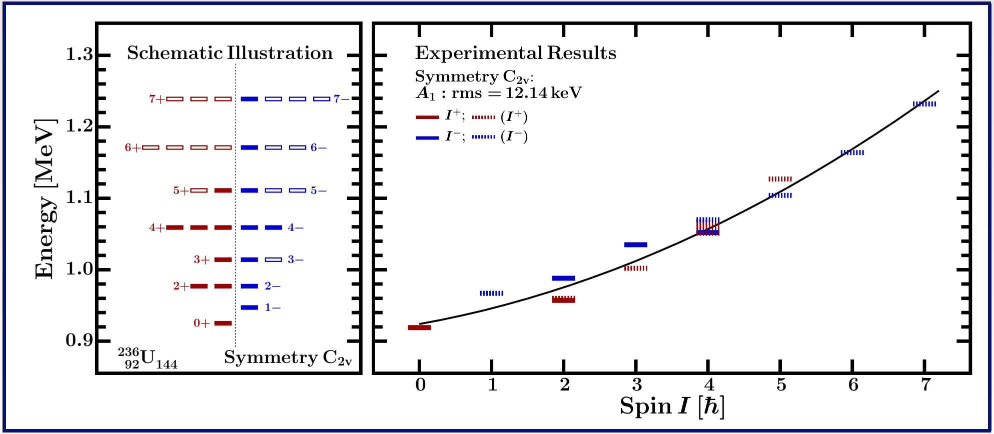

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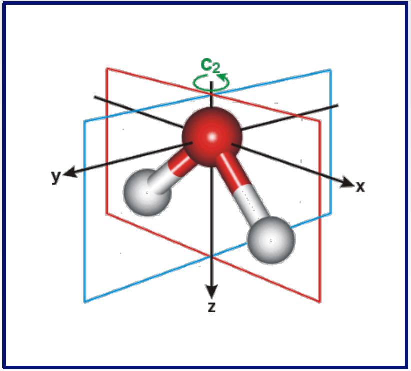
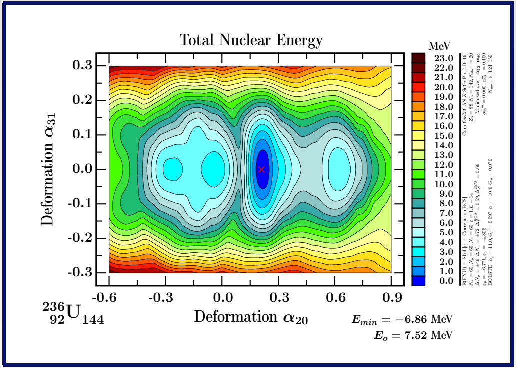
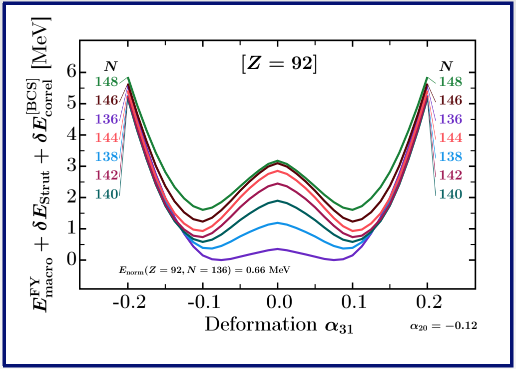
- H_2O has C_{2v}-symmetry:



Exotic Symmetries for ^{236}U – Suspects for C_{2v}





- We associate the prolate minimum at $\alpha_{20}^{th} \sim 0.25$ [r.m.s.(α_{20}^{exp}) = 0.2821(18)]$^{*)}$ with the ground-state,...
- ... and the oblate minimum at $\alpha_{20}^{th} \sim -0.12$ extended on α_{31} as the C_{2v} symmetry

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$^{*)}$ S. Raman, C. W. Nestor, JR., and P. Tikkanen
Atomic Data and Nuclear Data Tables, Vol. 78, No. 1, May 2001

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We will turn to the s o l u t i o n s
of the collective Schrödinger equation!!

Collective Schrödinger Equation

• Our group has developed*) new concepts of adiabaticity within collective model of Bohr and related approach to collective inertia tensor

• It follows that the collective energy operator is ($q^m \leftrightarrow \alpha_{\lambda,\mu}$, B-mass tensor)

$$\hat{H}_{\rm coll} = -\frac{\hbar^2}{2}\Delta + V(\alpha) \;\leftrightarrow\; \Delta \overset{df.}{=} \sum_{m,n=1}^{d} \frac{1}{\sqrt{|B|}} \frac{\partial}{\partial q^n}\left(\sqrt{|B|} B^{nm} \frac{\partial}{\partial q^m}\right).$$

with the resulting collective Schrödinger equation

$$\boxed{\hat{H}_{\rm coll}\Psi_{\rm coll} = E_{\rm coll}\Psi_{\rm coll}}$$

• All the details, illustrations, comparisons with experiment can be found in:

A New Approach to Adiabaticity Concepts in Collective Nuclear Motion: Impact for the Collective-Inertia Tensor and Comparisons with Experiment

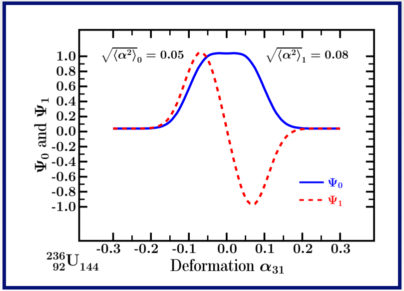
*) PHYSICAL REVIEW C 99, 041303(R) (2019)

D. Rouvel and J. Dudek

Collective Schrödinger Equation for C_{2v}

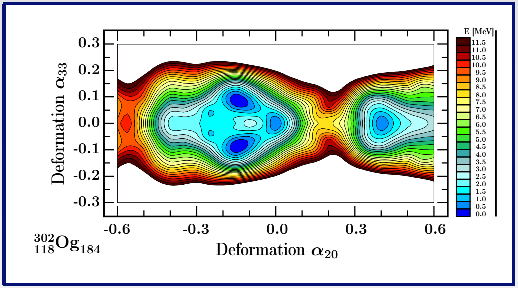
- The most probable α_{31} deformation $\leftrightarrow$ the so-called "dynamic equilibrium" $\leftrightarrow$ the most probable C_{2v}-symmetric shape

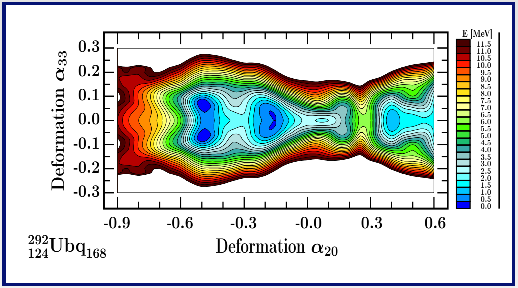
$$\alpha_{31}^{\text{dyn}} \;\leftrightarrow\; \langle \alpha_{31}^2 \rangle = \int \Psi^*(\alpha_{31})\alpha_{31}^2 \Psi(\alpha_{31}) d\alpha_{31}$$

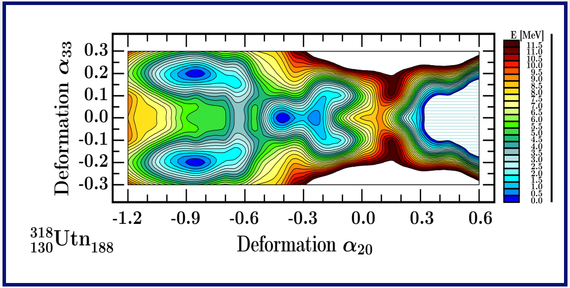


- Resulting dynamical equilibrium values are close to typical values of the secondary deformations such as the hexadecapole one reported in many nuclei

Theoretical Predictions for D_{3h} Symmetry $\Longleftrightarrow$ α_{33}







Interest/Impact of Our Research

- Just as an example, on of our latest publications on the theoretical predictions of D_{3h}-symmetry in super-heavy nuclei got 140 reads in 3 months

Islands of oblate hyperdeformed and superdeformed superheavy nuclei with D 3 h point group symmetry in competition with normal-deformed D 3 h states: "Archipelago" of D 3 h -symmetry islands

New | Article | Full-text available

May 2023 · 140 Reads

Jie Yang · Jerzy Dudek · Irene Dedes · [...] · Hua-Lei Wang

Add full-text

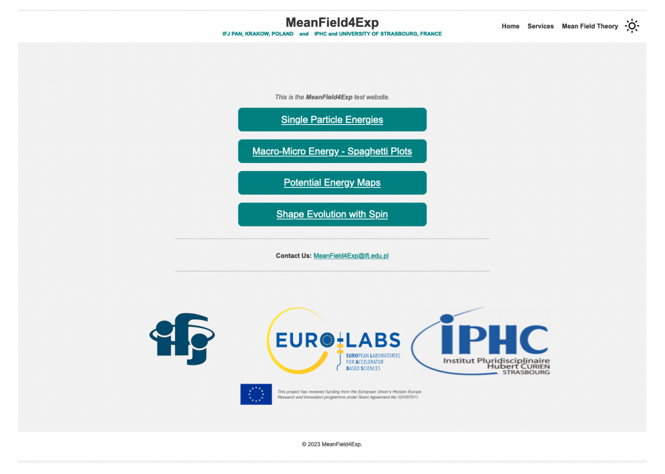
+12
Reads

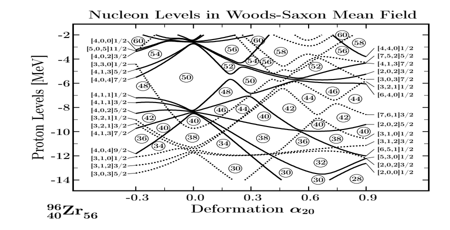
7 Full-text reads

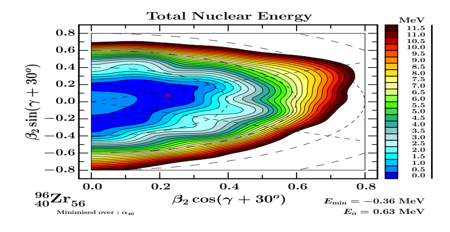
Current total: 140

EURO-LABS Project – MeanField4Exp

- Our Team is developing a website to improve the cooperation between nuclear physicists







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- We have as well presented predictions of D_{3h} symmetry in super-heavy nuclei competing with oblate hyper-deformation and super-deformation.

List of Publications for 2023

- *Islands of oblate hyper-deformed and super-deformed superheavy nuclei with D3h point group symmetry in competition with D_{3h} states: "Archipelago" of D_{3h}-symmetry islands;*

  Phys. Rev. C107, 054304 (2023)

- *From Exotic Mean-Field Symmetries to New Classes of Isomers in Atomic Nuclei*

  100 Years Anniversary of discovery of the Nuclear Isomerism;

  Invited Review, submitted to the European Physical Journal Special Topics, 2023

- *Unprecedented 7th-Order Multipole Components in Nuclear Equilibrium Deformations Induced by Tetrahedral Symmetry*

  submitted to Phys. Rev. C Letters, 2023, under processing

- *Combination of Stochastic and Group Theory Arguments for Identification of Molecular Symmetries in Subatomic Physics: Case of* C_{2v} *in* ^{236}U

  submitted to Phys. Rev. C Letters, 2023, under processing

- *Experimental Study of Competition Between Tetrahedral and Octahedral Symmetries in* ^{152}Sm *Nucleus: A New Evidence from a Designed Experiment*

  submitted to Phys. Rev. C, 2023, under processing